Problem 1. For each pair of expressions \((A, B)\) below, indicate whether \(A\) is \(O\), \(o\), \(\Omega\), \(\omega\), or \(\Theta\) of \(B\). Note that zero, one or more of these relations may hold for a given pair; list all correct ones.

\[
\begin{array}{ll}
A & B \\
(a) & \frac{n^4}{5} & 9n^3 + 1000n^2 \\
(b) & 2^n & n^{50} \\
(c) & \sqrt{n} & (\log n)^{12} \\
(d) & \sqrt{n} & n^{\cos(\pi n/8)} \\
(e) & 100^n & 10^{2n} \\
(f) & 2^{(n^2)} & 10^n \\
(g) & \log (n!) & n \log n
\end{array}
\]

Problem 2. Assume that your computer has special hardware that executes two comparisons in one step.

(a) Design an algorithm based on Merge Sort to sort \(n\) elements using this special hardware. (This is an upper bound.)

(b) Analyze your algorithm. Get the high order term (number of comparisons) exactly.

(c) Use decision trees to find a lower bound for sorting when using this special hardware.

(d) Compare your lower and upper bounds.

Problem 3.

(a) Assume you have an alphabet of letters from “i” to “o”. Illustrate the operation of radix sort on the following list of English words:

loin, mojo, noon, join, moon, kiln, milk, mono, jink, limo

(b) Use the words “jink” and “mojo” in an English sentence that indicates that you understand the meaning of both words.