Problem 1
You can solve this problem by the BFS algorithm.

Problem 2
You can solve this problem by dynamic programming. You have to fill a two dimensional array D[][].
Here D[n][i] represents the total number of different ways we can write n as the sum of powers of two which are less than or equal to $2^i$. Note that the second dimension is necessary. Also, note that the total size of D is $n \times \log n$.

Problem 3
You can solve this problem by the Dijkstra’s algorithm.