

CMSC 631
Program Analysis and Understanding
Spring 2013

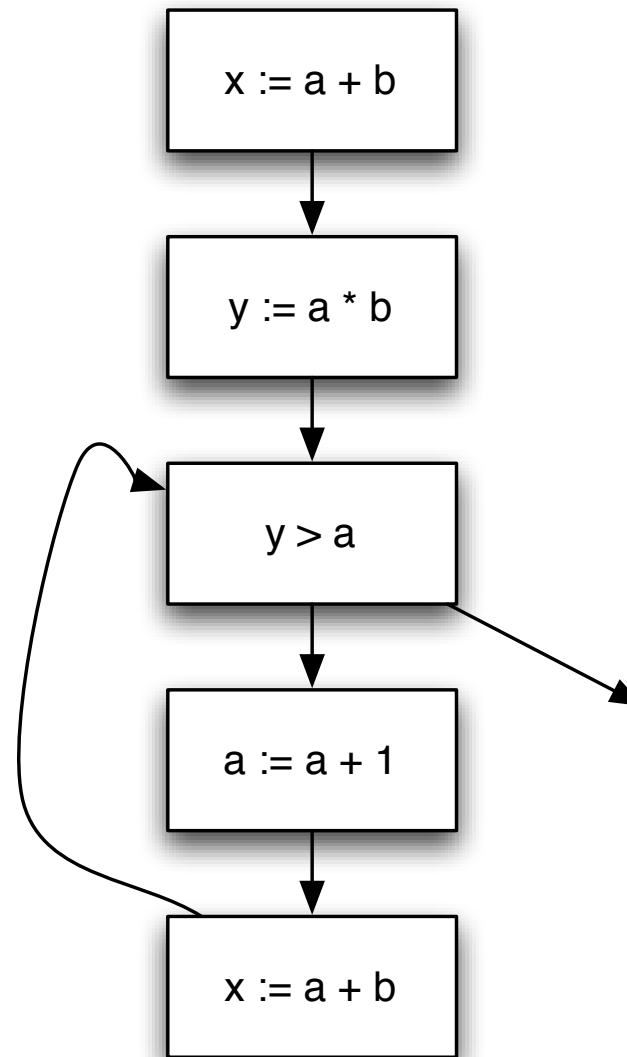
Data Flow Analysis

Data Flow Analysis

- A framework for proving facts about programs
- Reasons about lots of little facts
- Little or no interaction between facts
 - Works best on properties about *how* program computes
- Based on all paths through program
 - Including infeasible paths
- Operates on control-flow graphs, typically

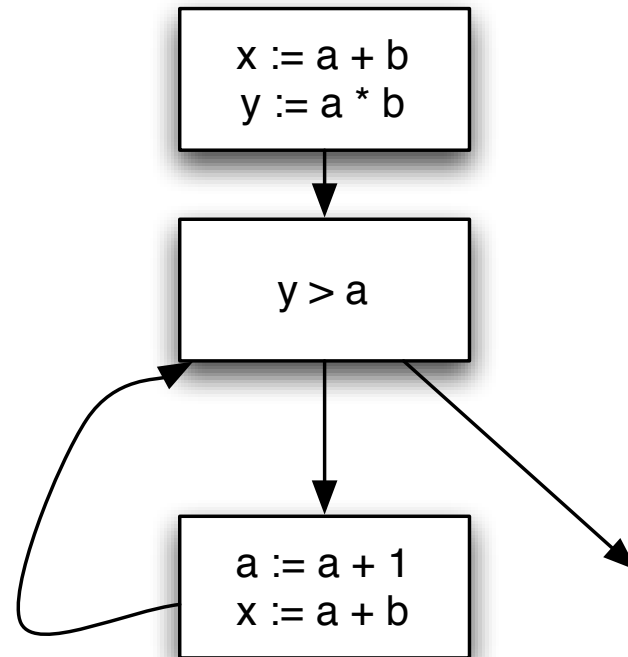
Control-Flow Graph Example

```
x := a + b;  
y := a * b;  
while (y > a) {  
    a := a + 1;  
    x := a + b  
}
```



Control-Flow Graph w/Basic Blocks

```
x := a + b;  
y := a * b;  
while (y > a + b) {  
    a := a + 1;  
    x := a + b  
}
```

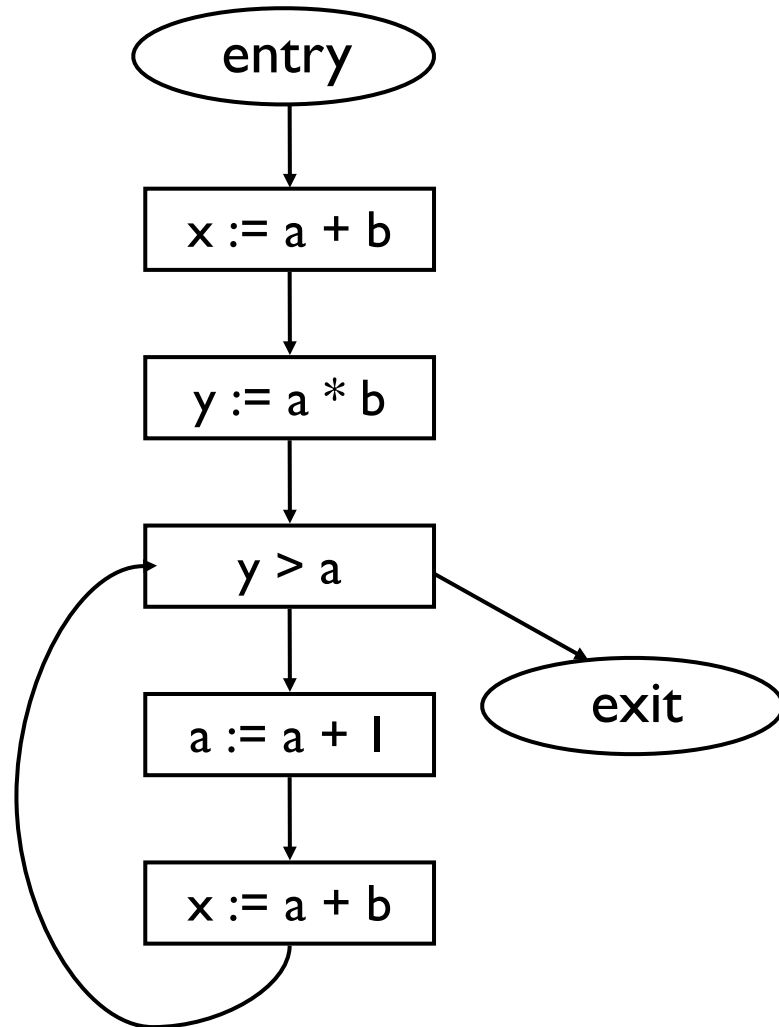


- Can lead to more efficient implementations
- But more complicated to explain, so...
 - We'll use single-statement blocks in lecture today

Example with Entry and Exit

```
x := a + b;  
y := a * b;  
while (y > a) {  
    a := a + 1;  
    x := a + b  
}
```

- All nodes without a (normal) predecessor should be pointed to by entry
- All nodes without a successor should point to exit



Notes on Entry and Exit

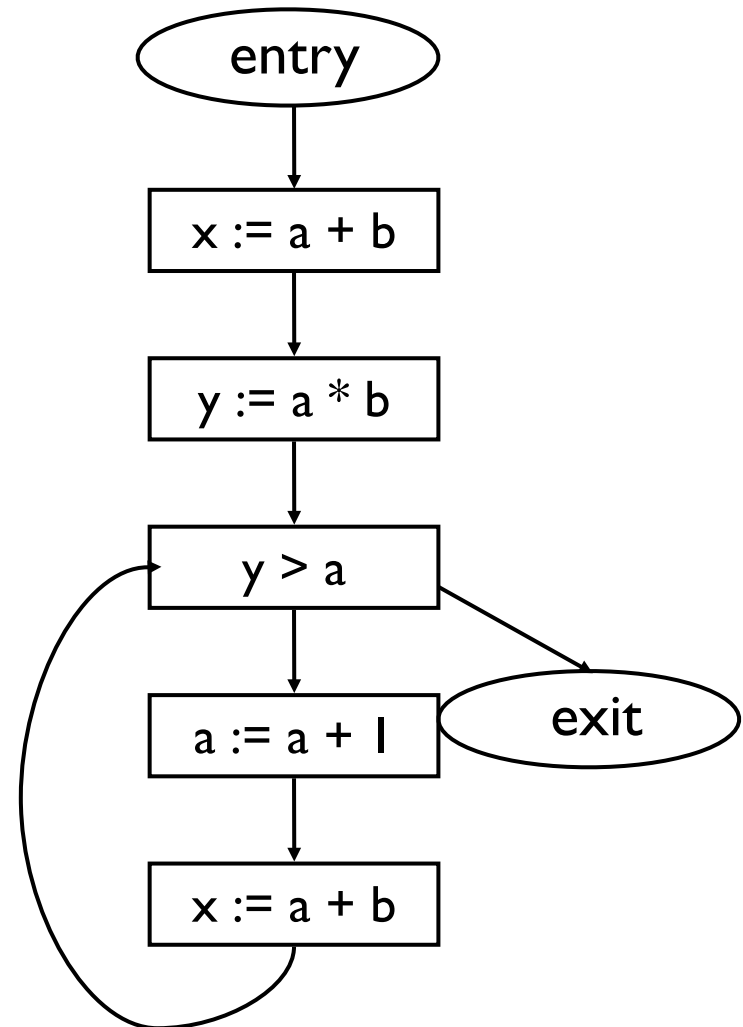
- Typically, we perform data flow analysis on a function body
- Functions usually have
 - A unique entry point
 - Multiple exit points
- So in practice, there can be multiple exit nodes in the CFG
 - For the rest of these slides, we'll assume there's only one
 - In practice, just treat all exit nodes the same way as if there's only one exit node

Available Expressions

- An expression e is available at program point p if
 - e is computed on every path to p , and
 - the value of e has not changed since the last time e was computed on the paths to p
- Optimization
 - If an expression is available, need not be recomputed
 - (At least, if it's still in a register somewhere)

Data Flow Facts

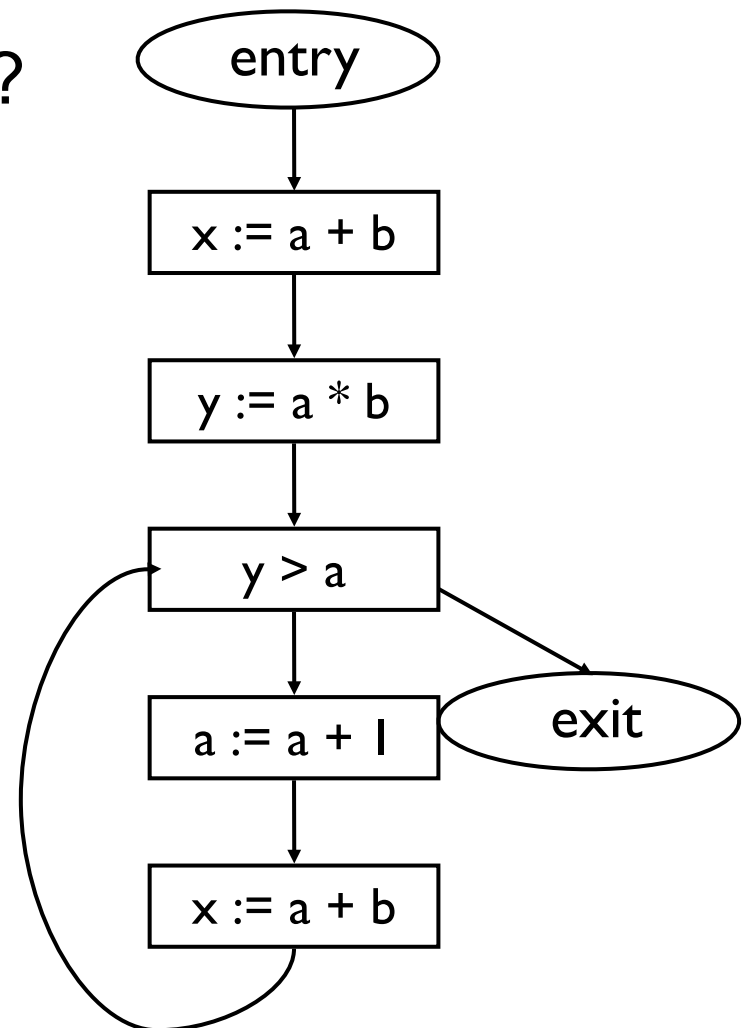
- Is expression e available?
- Facts:
 - $a + b$ is available
 - $a * b$ is available
 - $a + 1$ is available



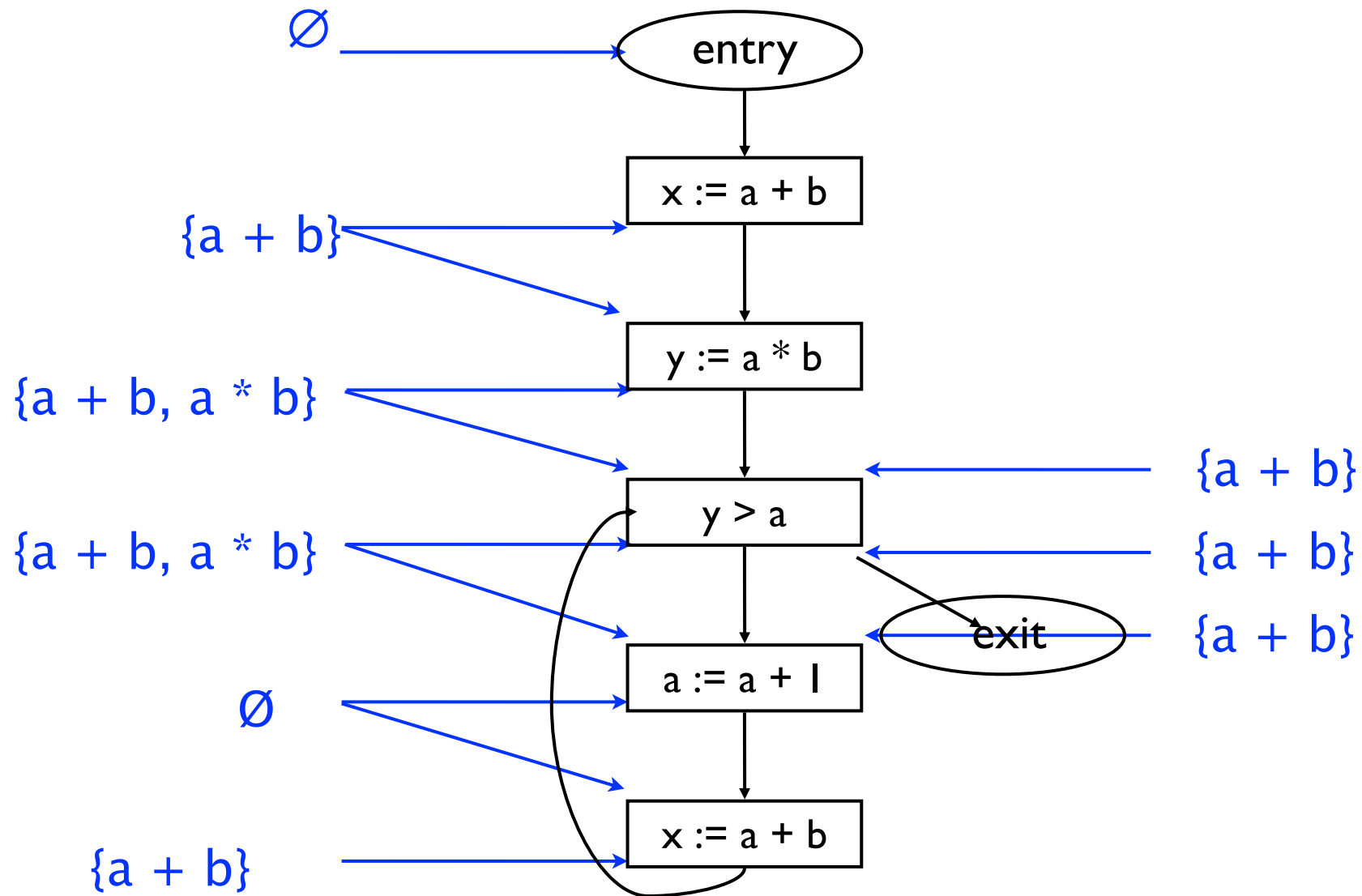
Gen and Kill

- What is the effect of each statement on the set of facts?

Stmt	Gen	Kill
$x := a + b$	$a + b$	
$y := a * b$	$a * b$	
$a := a + 1$		$a + 1,$ $a + b,$ $a * b$



Computing Available Expressions



Terminology

- A *joint point* is a program point where two branches meet
- Available expressions is a *forward must* problem
 - Forward = Data flow from **in** to **out**
 - Must = At join point, property must hold on all paths that are joined

Data Flow Equations

- Let s be a statement
 - $\text{succ}(s) = \{ \text{immediate successor statements of } s \}$
 - $\text{pred}(s) = \{ \text{immediate predecessor statements of } s \}$
 - $\text{in}(s) = \text{program point just before executing } s$
 - $\text{out}(s) = \text{program point just after executing } s$
- $\text{in}(s) = \bigcap_{s' \in \text{pred}(s)} \text{out}(s')$
- $\text{out}(s) = \text{gen}(s) \cup (\text{in}(s) - \text{kill}(s))$
 - Note: These are also called *transfer functions*

Liveness Analysis

- A variable v is *live* at program point p if
 - v will be used on some execution path originating from p ...
 - before v is overwritten
- Optimization
 - If a variable is not live, no need to keep it in a register
 - If variable is dead at assignment, can eliminate assignment

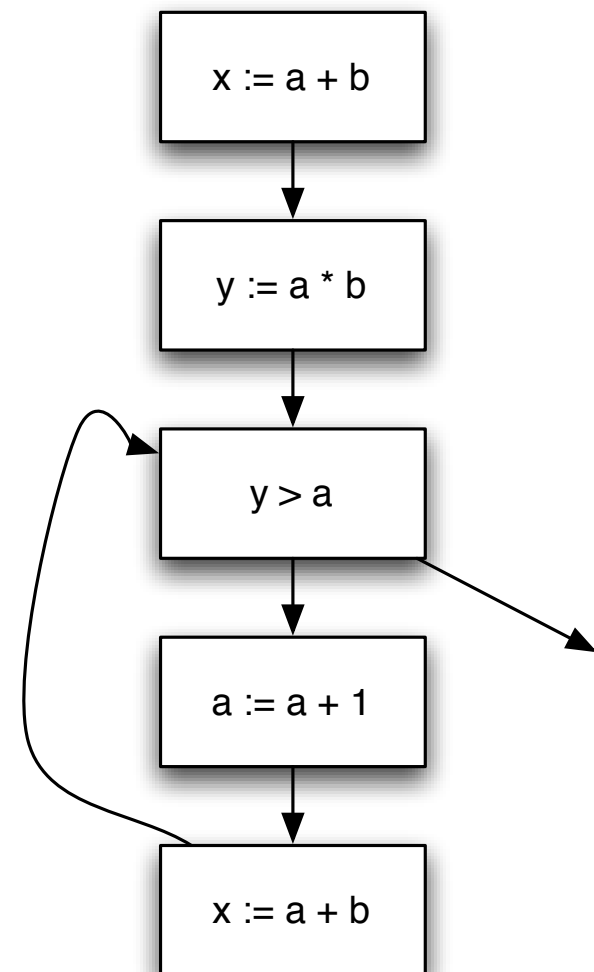
Data Flow Equations

- Available expressions is a forward must analysis
 - Data flow propagate in same dir as CFG edges
 - Expr is available only if available on all paths
- Liveness is a *backward may* problem
 - To know if variable live, need to look at future uses
 - Variable is live if used on some path
- $$\text{out}(s) = \bigcup_{s' \in \text{succ}(s)} \text{in}(s')$$
- $$\text{in}(s) = \text{gen}(s) \cup (\text{out}(s) - \text{kill}(s))$$

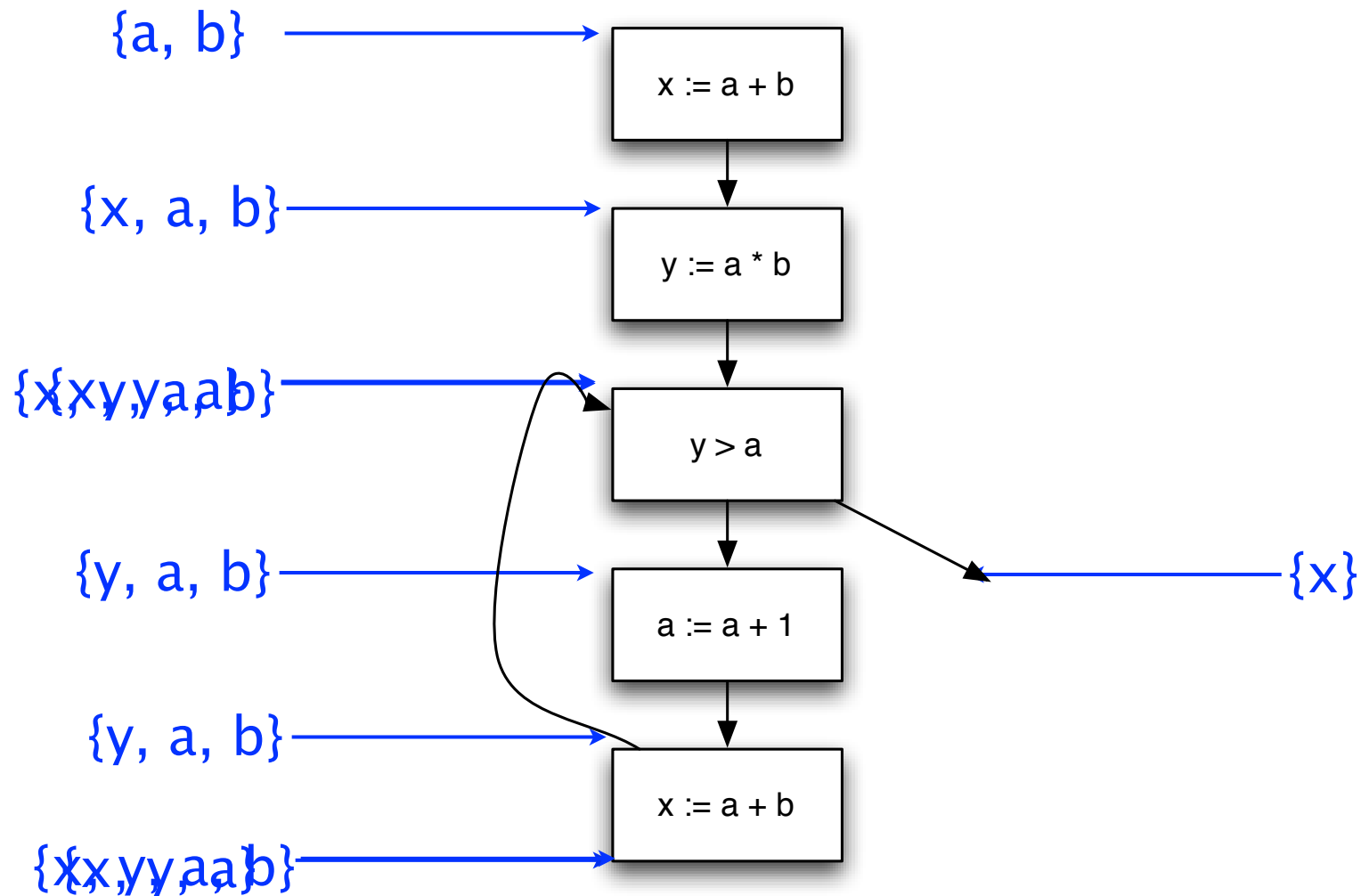
Gen and Kill

- What is the effect of each statement on the set of facts?

Stmt	Gen	Kill
$x := a + b$	a, b	x
$y := a * b$	a, b	y
$y > a$	a, y	
$a := a + 1$	a	a



Computing Live Variables



Very Busy Expressions

- An expression *e* is *very busy* at point *p* if
 - On every path from *p*, expression *e* is evaluated before the value of *e* is changed
- Optimization
 - Can hoist very busy expression computation
- What kind of problem?
 - Forward or backward? *backward*
 - May or must? *must*

Reaching Definitions

- A *definition* of a variable v is an assignment to v
- A definition of variable v reaches point p if
 - There is no intervening assignment to v
- Also called def-use information
- What kind of problem?
 - Forward or backward? forward
 - May or must? may

Space of Data Flow Analyses

	May	Must
Forward	Reaching definitions	Available expressions
Backward	Live variables	Very busy expressions

- Most data flow analyses can be classified this way
 - A few don't fit: bidirectional analysis
- Lots of literature on data flow analysis

Solving data flow equations

- Let's start with forward may analysis
 - Dataflow equations:
 - $\text{in}(s) = \bigcup_{s' \in \text{pred}(s)} \text{out}(s')$
 - $\text{out}(s) = \text{gen}(s) \cup (\text{in}(s) - \text{kill}(s))$
- Need algorithm to compute **in** and **out** at each stmt
- Key observation: **out(s)** is *monotonic* in **in(s)**
 - **gen(s)** and **kill(s)** are fixed for a given s
 - If, during our algorithm, **in(s)** grows, then **out(s)** grows
 - Furthermore, **out(s)** and **in(s)** have max size
- Same with **in(s)**
 - in terms of **out(s')** for predecessors **s'**

Solving data flow equations (cont'd)

- Idea: fixpoint algorithm
 - Set $\text{out}(\text{entry})$ to emptyset
 - E.g., we know no definitions reach the entry of the program
 - Initially, assume $\text{in}(s)$, $\text{out}(s)$ empty everywhere else, also
 - Pick a statement s
 - Compute $\text{in}(s)$ from predecessors' out 's
 - Compute new $\text{out}(s)$ for s
 - Repeat until nothing changes
- Improvement: use a worklist
 - Add statements to worklist if their $\text{in}(s)$ might change
 - Fixpoint reached when worklist is empty

Forward May Data Flow Algorithm

```
out(entry) =  $\emptyset$ 
for all other statements  $s$ 
  out( $s$ ) =  $\emptyset$ 
 $W = \text{all statements}$  // worklist
while  $W$  not empty
  take  $s$  from  $W$ 
  in( $s$ ) =  $\cup_{s' \in \text{pred}(s)} \text{out}(s')$ 
  temp = gen( $s$ )  $\cup$  (in( $s$ ) - kill( $s$ ))
  if temp  $\neq$  out( $s$ ) then
    out( $s$ ) = temp
     $W := W \cup \text{succ}(s)$ 
  end
end
```

Generalizing

	May	Must
Forward	$\text{in}(s) = \bigcup_{s' \in \text{pred}(s)} \text{out}(s')$ $\text{out}(s) = \text{gen}(s) \cup (\text{in}(s) - \text{kill}(s))$ $\text{out}(\text{entry}) = \emptyset$ $\text{initial out elsewhere} = \emptyset$	$\text{in}(s) = \bigcap_{s' \in \text{pred}(s)} \text{out}(s')$ $\text{out}(s) = \text{gen}(s) \cup (\text{in}(s) - \text{kill}(s))$ $\text{out}(\text{entry}) = \emptyset$ $\text{initial out elsewhere} = \{\text{all facts}\}$
Backward	$\text{out}(s) = \bigcup_{s' \in \text{succ}(s)} \text{in}(s')$ $\text{in}(s) = \text{gen}(s) \cup (\text{out}(s) - \text{kill}(s))$ $\text{in}(\text{exit}) = \emptyset$ $\text{initial in elsewhere} = \emptyset$	$\text{out}(s) = \bigcap_{s' \in \text{succ}(s)} \text{in}(s')$ $\text{in}(s) = \text{gen}(s) \cup (\text{out}(s) - \text{kill}(s))$ $\text{in}(\text{exit}) = \emptyset$ $\text{initial out elsewhere} = \{\text{all facts}\}$

Forward Analysis

```
out(entry) =  $\emptyset$ 
for all other statements  $s$ 
  out( $s$ ) =  $\emptyset$ 
 $W = \text{all statements}$  // worklist
while  $W$  not empty
  take  $s$  from  $W$ 
  in( $s$ ) =  $\cup_{s' \in \text{pred}(s)} \text{out}(s')$ 
  temp = gen( $s$ )  $\cup$  (in( $s$ ) - kill( $s$ ))
  if temp  $\neq$  out( $s$ ) then
    out( $s$ ) = temp
     $W := W \cup \text{succ}(s)$ 
  end
end
```

May

```
out(entry) =  $\emptyset$ 
for all other statements  $s$ 
  out( $s$ ) = all facts
 $W = \text{all statements}$ 
while  $W$  not empty
  take  $s$  from  $W$ 
  in( $s$ ) =  $\cap_{s' \in \text{pred}(s)} \text{out}(s')$ 
  temp = gen( $s$ )  $\cup$  (in( $s$ ) - kill( $s$ ))
  if temp  $\neq$  out( $s$ ) then
    out( $s$ ) = temp
     $W := W \cup \text{succ}(s)$ 
  end
end
```

Must

Backward Analysis

```
in(exit) =  $\emptyset$ 
for all other statements s
  in(s) =  $\emptyset$ 
W = all statements
while W not empty
  take s from W
  out(s) =  $\cup_{s' \in \text{succ}(s)} \text{in}(s')$ 
  temp =  $\text{gen}(s) \cup (\text{out}(s) - \text{kill}(s))$ 
  if temp  $\neq$  in(s) then
    in(s) = temp
    W := W  $\cup$  pred(s)
  end
end
```

May

```
in(exit) =  $\emptyset$ 
for all other statements s
  in(s) = all facts
W = all statements
while W not empty
  take s from W
  out(s) =  $\cap_{s' \in \text{succ}(s)} \text{in}(s')$ 
  temp =  $\text{gen}(s) \cup (\text{out}(s) - \text{kill}(s))$ 
  if temp  $\neq$  in(s) then
    in(s) = temp
    W := W  $\cup$  pred(s)
  end
end
```

Must

Practical Implementation

- Represent set of facts as bit vector
 - Fact_i represented by bit i
 - Intersection = bitwise and, union = bitwise or, etc
- “Only” a constant factor speedup
 - But very useful in practice

Generalizing Further

- Observe $\text{out}(s)$ is a *function* of $\text{out}(s')$ for preds s'

$$\text{out}(s) = \text{gen}(s) \cup ((\bigcup_{s' \in \text{pred}(s)} \text{out}(s')) - \text{kill}(s))$$

- We can define other kinds of functions, to compute other kinds of information using dataflow analysis!
- Example: constant propagation
 - Facts — variable x has value n (at this program point)
 - Not quite gen/kill:

```
/* facts: a = 1, b = 2 */
```

```
x = a + b
```

```
/* facts: a = 1, b = 2, x = 3 */
```

- Fact that x is 3 not determined syntactically by statement
- So, how can we use data flow analysis to handle this case?

Partial Orders

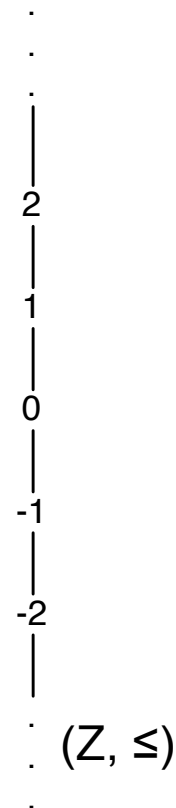
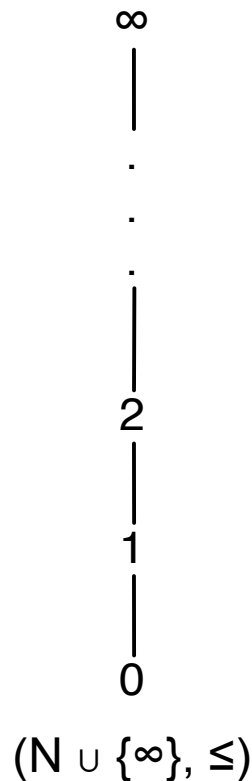
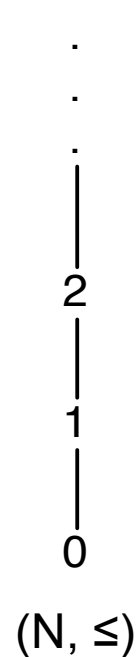
- To generalize data flow analysis, need to introduce two mathematical structures:
 - Partial orders
 - Lattices
- A *partial order* (p.o.) is a pair $(P, \leq)$ such that
 - $\leq \subseteq P \times P$
 - $\leq$ is reflexive: $x \leq x$
 - $\leq$ is anti-symmetric: $x \leq y$ and $y \leq x \Rightarrow x = y$
 - $\leq$ is transitive: $x \leq y$ and $y \leq z \Rightarrow x \leq z$

Examples

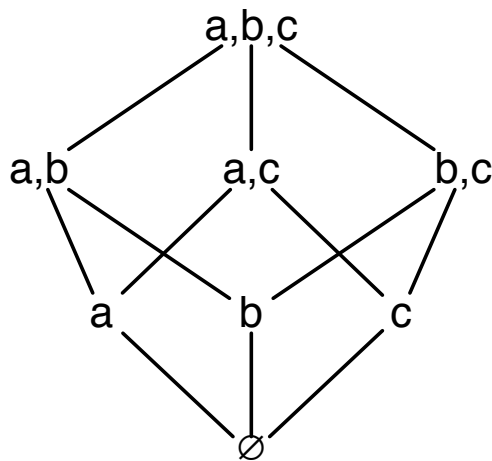
- $(\mathbb{N}, \leq)$
 - Natural numbers with standard inequality
- $(\mathbb{N} \cup \{\infty\}, \leq)$
 - Natural numbers plus infinity, with standard inequality
- $(\mathbb{Z}, \leq)$
 - Integers with standard inequality
- For any set S , $(2^S, \subseteq)$
 - The *powerset partial order*
- For any set S , $(S, =)$
 - The *discrete partial order*
- A 2-crown $(\{a,b,c,d\}, \{a < c, a < d, b < c, b < d\})$

Drawing Partial Orders

- We can write partial orders as graphs using the following conventions
 - Nodes are elements of the p.o.
 - Edge from element lower on page to high on page means lower element is strictly less than higher element



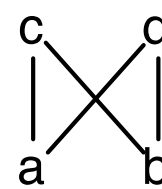
Drawing Partial Orders (cont'd)



$(\{a, b, c\}, \subseteq)$

$a \quad b \quad c$

$(\{a, b, c\}, =)$



2-crown

Meet and Join Operations

- $\sqcap$ is the *meet* or *greatest lower bound* operation:
 - $x \sqcap y \leq x$ and $x \sqcap y \leq y$
 - if $z \leq x$ and $z \leq y$ then $z \leq x \sqcap y$
- $\sqcup$ is the *join* or *least upper bound* operation:
 - $x \leq x \sqcup y$ and $y \leq x \sqcup y$
 - if $x \leq z$ and $y \leq z$ then $x \sqcup y \leq z$

Examples

- $(\mathbb{N}, \leq)$, $(\mathbb{N} \cup \{\infty\}, \leq)$, $(\mathbb{Z}, \leq)$
 - $\sqcap = \min$, $\sqcup = \max$
- For any set S , $(2^S, \subseteq)$
 - $\sqcap = \cap$, $\sqcup = \cup$
- For any set S , $(S, =)$
 - $\sqcap$ and $\sqcup$ only defined when element is the same
- A 2-crown $(\{a,b,c,d\}, \{a < c, a < d, b < c, b < d\})$
 - $a \sqcup b$ and $c \sqcap d$ undefined

Lattices

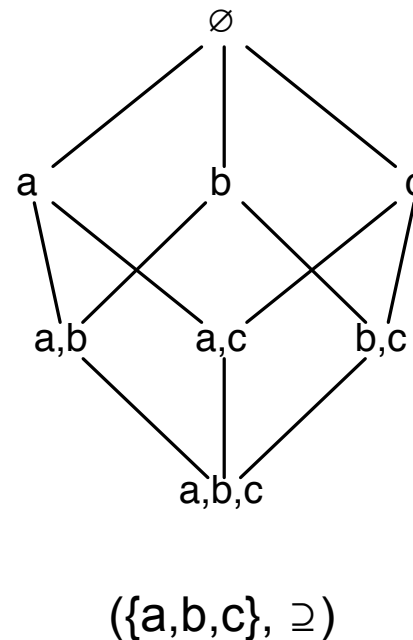
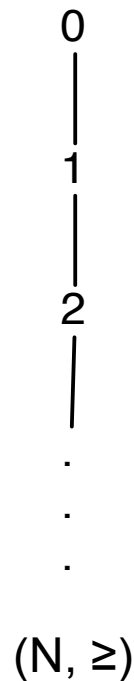
- A p.o. is a *lattice* if $\sqcap$ and $\sqcup$ are defined on any two elements
 - A partial order is a *complete lattice* if $\sqcap$ and $\sqcup$ are defined on any set
- A lattice has unique elements $\perp$ (“bottom”) and $\top$ (“top”) such that
 - $x \sqcap \perp = \perp$ $x \sqcup \perp = x$
 - $x \sqcap \top = x$ $x \sqcup \top = \top$
- In a lattice,
 - $x \leq y$ iff $x \sqcap y = x$
 - $x \leq y$ iff $x \sqcup y = y$

Examples

- $(\mathbb{N}, \leq)$
 - $\perp = 0$, $\top$ undefined; is a lattice, but not a complete lattice
- $(\mathbb{N} \cup \{\infty\}, \leq)$
 - $\perp = 0$, $\top = \infty$; is a complete lattice
- $(\mathbb{Z}, \leq)$
 - $\perp$, $\top$ undefined; is a lattice, but not a complete lattice
- For any set S , $(2^S, \subseteq)$
 - $\perp = \emptyset$, $\top = S$, is a complete lattice
- For any set S , $(S, =)$
 - $\perp$, $\top$ undefined; not a lattice
- A 2-crown $(\{a,b,c,d\}, \{a < c, a < d, b < c, b < d\})$
 - $\perp$, $\top$ undefined; not a lattice

Flipping a Lattice

- Lemma: If $(P, \leq)$ is a lattice, then $(P, \lambda xy. y \leq x)$ is also a lattice
 - I.e., if we flip the sense of $\leq$, we still have a lattice
- Examples:



Cross-product lattice

- Lemma: Suppose $(P, \leq_1)$ and $(Q, \leq_2)$ are lattices. Then $(P \times Q, \leq)$ is also a lattice, where
 - $(p,q) \leq (p', q')$ iff $p \leq_1 p'$ and $q \leq_2 q'$
 - (Can also take cross product of more than 2 lattices, in the natural way)

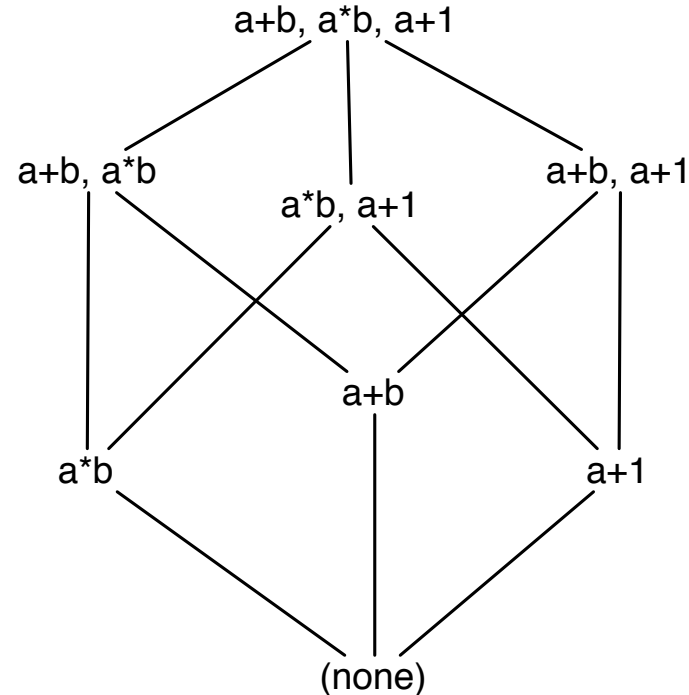
• Examples:

$$\begin{array}{c} b \\ | \\ a \end{array} \times \begin{array}{c} d \\ | \\ c \end{array} = \begin{array}{ccccc} & & (b,d) & & \\ & \swarrow & & \searrow & \\ (b,c) & & & & (a,d) \\ & \swarrow & & \searrow & \\ & & (a,c) & & \end{array}$$

$$\begin{array}{ccccc} & & (b,d) & & \\ & \swarrow & & \searrow & \\ (b,c) & & & & (a,d) \\ & \swarrow & & \searrow & \\ & & (a,c) & & \end{array} \times \begin{array}{c} f \\ | \\ e \end{array} = \begin{array}{ccccc} & & (b,d,f) & & \\ & \swarrow & & \searrow & \\ (b,c,f) & & (a,d,f) & & (b,d,e) \\ & \swarrow & & \searrow & \\ (a,c,f) & & (b,c,e) & & (a,d,e) \\ & \swarrow & & \searrow & \\ & & (a,c,e) & & \end{array}$$

Data Flow Facts and Lattices

- Sets of dataflow facts form the powerset lattice
 - Example: Available expressions



Transfer Functions

- Recall this step from *forward must* analysis:

$$\text{in}(s) := \bigcap_{s' \in \text{pred}(s)} \text{out}(s')$$

$$\text{temp} := \text{gen}(s) \cup (\text{in}(s) - \text{kill}(s))$$

- Let's recast this in terms of powerset lattice

- $\bigcap$ is $\sqcap$ in the lattice
- $\text{gen}(s)$, $\text{kill}(s)$ are fixed
 - So temp is a function of $\text{in}(s)$
- Putting this together:

$$\text{in}(s) := \bigcap_{s' \in \text{pred}(s)} \text{out}(s')$$

$$\text{temp} := f_s(\text{in}(s))$$

$$\text{where } f_s(x) = \text{gen}(s) \cup (x - \text{kill}(s))$$

f_s is a
transfer function

Forward May Analysis

- What about forward may analysis?

$$\text{in}(s) := \bigcup_{s' \in \text{pred}(s)} \text{out}(s')$$

$$\text{temp} := \text{gen}(s) \cup (\text{in}(s) - \text{kill}(s))$$

- We can just use the flipped powerset lattice
 - $\bigcup$ is $\sqcap$ is that lattice
- So we get the same equations

$$\text{in}(s) := \bigcap_{s' \in \text{pred}(s)} \text{out}(s')$$

$$\text{temp} := f_s(\text{in}(s))$$

$$\text{where } f_s(x) = \text{gen}(s) \cup x - \text{kill}(s)$$

- Same idea for must/may backward analysis
 - But still separate from forward analysis

Initial Facts

- Recall also from *forward must* analysis:

$$\text{out}(\text{entry}) = \emptyset$$

$$\text{initial out elsewhere} = \{\text{all facts}\}$$

- Values of these in lattice terms depends on analysis
 - Available expressions
 - $\text{out}(\text{entry})$ is the same as $\perp$
 - initial out elsewhere is the same as $\top$
 - Reaching definitions (with $\leq$ as $\supseteq$)
 - $\text{out}(\text{entry})$ is $\emptyset$ which is $\top$ in this lattice (flipped powerset)
 - initial out elsewhere is also $\top$

Data Flow Analysis, over Lattices

```
out(entry) = (as given)
for all other statements s
    out(s) =  $\top$ 
W = all statements    // worklist
while W not empty
    take s from W
    in(s) =  $\sqcap_{s' \in \text{pred}(s)} \text{out}(s')$ 
    temp =  $f_s(\text{in}(s))$ 
    if temp  $\neq$  out(s) then
        out(s) = temp
        W := W  $\cup$  succ(s)
    end
end
```

Forward

(Red = varies by analysis)

```
in(exit) = (as given)
for all other statements s
    in(s) =  $\top$ 
W = all statements
while W not empty
    take s from W
    out(s) =  $\sqcap_{s' \in \text{succ}(s)} \text{in}(s')$ 
    temp =  $f_s(\text{out}(s))$ 
    if temp  $\neq$  in(s) then
        in(s) = temp
        W := W  $\cup$  pred(s)
    end
end
```

Backward

DFA over Lattices, cont'd

- A dataflow analysis is defined by 4 things:
 - Forward or backward
 - The lattice
 - Data flow facts
 - $\sqcap$ operation
 - In terms of gen/kill dfa, this specifies may or must
 - $\top$ value
 - In terms of gen/kill dfa, this specifies the initial facts assumed at each statement
 - Transfer functions
 - In terms of gen/kill dfa, this defines gen and kill
 - Facts at entry (for forward) or exit (for backward)
 - In terms of gen/kill dfa, this defines set of facts for entry or exit node

Four Analyses as Lattices ($P, \leq$)

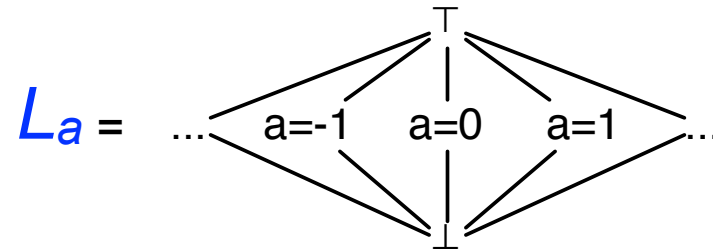
- Available expressions
 - Forward analysis
 - P = sets of expressions
 - $S1 \sqcap S2 = S1 \cap S2$
 - Top = set of all expressions
 - Entry facts = $\emptyset$ = no expressions available at entry
- Reaching Definitions
 - Forward analysis
 - P = set of definitions (assignment statements)
 - $S1 \sqcap S2 = S1 \cup S2$
 - Top = empty set
 - Entry facts = $\emptyset$ = no definitions reach entry

Four Analyses as Lattices ($P, \leq$)

- Very busy expressions
 - Backward analysis
 - P = sets of expressions
 - $S1 \sqcap S2 = S1 \cap S2$
 - Top = set of all expressions
 - Exit facts = $\emptyset$ = no expressions busy at exit
- Live variables
 - Backward analysis
 - P = set of variables
 - $S1 \sqcap S2 = S1 \cup S2$
 - Top = empty set
 - Exit facts = $\emptyset$ = no variables live at exit

Constant Propagation

- Idea: maintain possible value of each variable



- $\top$ = initial value = haven't seen assignment to a yet
- $\perp$ = multiple different possible values for a
- DFA definition:
 - Forward analysis
 - Lattice = $L_a \times L_b \times \dots$ (for all variables in program)
 - I.e., maintain one possible value of each variable
 - Initial facts (at entry) = $\top$ (variables all unassigned)

Monotonicity and Desc. Chain

- A function f on a partial order is *monotonic* (or *order preserving*) if

$$x \leq y \Rightarrow f(x) \leq f(y)$$

- Examples
 - $\lambda x.x+1$ on partial order $(\mathbb{Z}, \leq)$ is monotonic
 - $\lambda x.-x$ on partial order $(\mathbb{Z}, \leq)$ is not monotonic
- Transfer functions in gen/kill DFA are monotonic
 - $\text{temp} = \text{gen}(s) \cup (\text{in}(s) - \text{kill}(s))$
 - Holds because $\text{gen}(s)$ and $\text{kill}(s)$ are fixed
 - Thus, if we shrink $\text{in}(s)$, temp can only shrink
- A *descending chain* in a lattice is a sequence
 - $x_0 \sqsupset x_1 \sqsupset x_2 \sqsupset \dots$
 - *Height* of lattice = length of longest descending chain

Monotonicity and Transfer Fns

- If f_s is monotonic, how often can we apply this step?

$$\begin{aligned} \text{in}(s) &= \bigcap_{s' \in \text{pred}(s)} \text{out}(s') \\ \text{temp} &= f_s(\text{in}(s)) \end{aligned}$$

- Claim: $\text{out}(s)$ only shrinks
 - Proof: $\text{out}(s)$ starts out as $\top$
 - Assume $\text{out}(s')$ shrinks for all predecessors s' of s
 - Then $\bigcap_{s' \in \text{pred}(s)} \text{out}(s')$ shrinks
 - Since f_s monotonic, $f_s(\bigcap_{s' \in \text{pred}(s)} \text{out}(s'))$ shrinks

Termination

- Suppose we have a DFA with
 - Finite height lattice
 - Monotonic transfer functions

} Both hold for gen/kill
probs and constant prop
- Then, at every step in DFA we
 - Remove a statement from the worklist, and/or
 - Strictly decrease some dataflow fact at a program point
 - (By monotonicity)
 - Only add new statements to worklist after strict decrease
 - $\Rightarrow$ termination! (by finite height)
- Moreover, must terminate in $O(nk)$ time
 - n = # of statements in program
 - k = height of lattice
 - (assumes meet operation takes $O(1)$ time)

Fixpoints

- We always start with $\top$
 - E.g., every expr is available, no defs reach this point
 - Most optimistic assumption
 - Strongest possible hypothesis
 - = true of fewest number of states
- Revise as we encounter contradictions
 - Always move down in the lattice (with meet)
- Result: A *greatest fixpoint* solution of the data flow equations

Least vs. Greatest Fixpoints

- Dataflow tradition: Start with $\top$, use $\sqcap$
 - Computes a greatest fixpoint
 - Technically, rather than a lattice, we only need a
 - *finite height meet semilattice with top*
 - (*meet semilattice* = $a \sqcap b$ defined on any a, b but $a \sqcup b$ may not be defined)
- Denotational semantics trad.: Start with $\perp$, use $\sqcup$
 - Computes least fixpoint
- So, direction of DFA may depend on community author comes from...

Distributive Dataflow Problems

- A monotonic transfer function f also satisfies

$$f(x \sqcap y) \leq f(x) \sqcap f(y)$$

- Proof: By monotonicity, $f(x \sqcap y) \leq f(x)$ and $f(x \sqcap y) \leq f(y)$, i.e., $f(x \sqcap y)$ is a lower bound of $f(x)$ and $f(y)$. But then since $\sqcap$ is the greatest lower bound, $f(x \sqcap y) \leq f(x) \sqcap f(y)$.

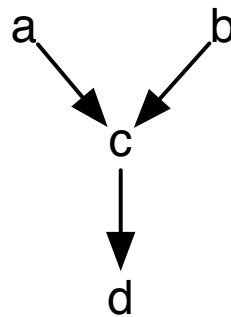
- A transfer function f is *distributive* if it satisfies

$$f(x \sqcap y) = f(x) \sqcap f(y)$$

- Notice this is stronger than monotonicity

Benefit of Distributivity

- Suppose we have the following CFG with four statements, a, b, c, d , with transfer fns f_a, f_b, f_c, f_d



- Then joins lose no information!
 - $\text{out}(a) = f_a(\text{in}(a))$
 - $\text{out}(b) = f_b(\text{in}(b))$
 - $\text{out}(c) = f_c(\text{out}(a) \sqcap \text{out}(b)) = f_c(f_a(\text{in}(a)) \sqcap f_b(\text{in}(b))) = f_c(f_a(\text{in}(a))) \sqcap f_c(f_b(\text{in}(b)))$
 - $\text{out}(d) = f_d(\text{out}(c)) = f_d(f_c(f_a(\text{in}(a))) \sqcap f_c(f_b(\text{in}(b)))) = f_d(f_c(f_a(\text{in}(a)))) \sqcap f_d(f_c(f_b(\text{in}(b))))$

Accuracy of Data Flow Analysis

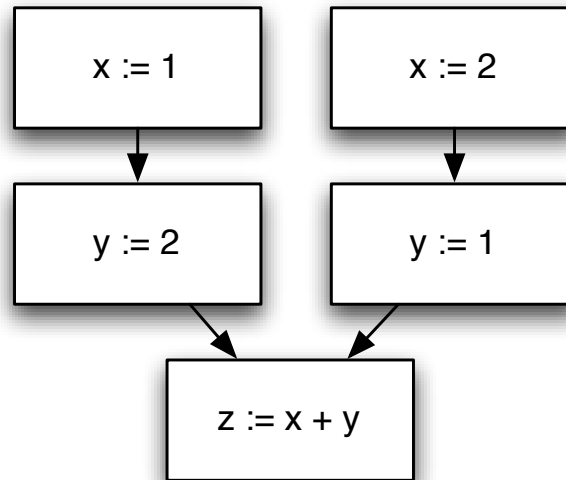
- Ideally, we would like to compute the *meet over all paths* (MOP) solution:
 - If p is a path through the CFG, let f_p be the composition of transfer functions for the statements along p
 - Let $\text{path}(s)$ be the set of paths from the entry to s
 - Define
$$\text{MOP}(s) = \sqcap_{p \in \text{path}(s)} f_p(\top)$$
 - I.e., $\text{MOP}(s)$ is the set of dataflow facts if we separately apply the transfer functions along every path (assuming $\top$ is the initial value at the entry) and then apply $\sqcap$ to the result
 - This is the best we could possibly do if we want one data flow fact per program point and we ignore conditional tests along the path
- If a data flow problem is distributive, then solving the data flow equations in the standard way yields the MOP solution!

What Problems are Distributive?

- Analyses of *how* the program computes
 - Live variables
 - Available expressions
 - Reaching definitions
 - Very busy expressions
- All gen/kill problems are distributive

A Non-Distributive Example

- Constant propagation



- $\{x=1, y=2\} \sqcap \{x=2, y=1\} = \{x=\perp, y=\perp\}$
- The join at $\text{in}(z:=x+y)$ loses information
- But in fact, $z=3$ every time we run this program
- In general, analysis of *what* the program computes is not distributive

Basic Blocks

- Recall a *basic block* is a sequence of statements s.t.
 - No statement except the last in a branch
 - There are no branches to any statement in the block except the first
- In some data flow implementations,
 - Compute gen/kill for each basic block as a whole
 - Compose transfer functions
 - Store only in/out for each basic block
 - Typical basic block ~5 statements
 - At least, this used to be the case...

Order Matters

- Assume forward data flow problem
 - Let $G = (V, E)$ be the CFG
 - Let k be the height of the lattice
- If G acyclic, visit in topological order
 - Visit head before tail of edge
- Running time $O(|E|)$
 - No matter what size the lattice

Order Matters — Cycles

- If G has cycles, visit in reverse postorder
 - Order from depth-first search
 - (Reverse for backward analysis)
- Let $Q = \max \#$ back edges on cycle-free path
 - Nesting depth
 - Back edge is from node to ancestor in DFS tree
- If $\forall x. f(x) \leq x$ (sufficient, but not necessary), then running time is $O((Q+1)|E|)$
 - Proportional to structure of CFG rather than lattice

Flow-Sensitivity

- Data flow analysis is *flow-sensitive*
 - The order of statements is taken into account
 - I.e., we keep track of facts per program point
- Alternative: *Flow-insensitive* analysis
 - Analysis the same regardless of statement order
 - Standard example: types
 - `/* x : int */ x := ... /* x : int */`

Data Flow Analysis and Functions

- What happens at a function call?
 - Lots of proposed solutions in data flow analysis literature
- In practice, only analyze one procedure at a time
- Consequences
 - Call to function kills all data flow facts
 - May be able to improve depending on language, e.g., function call may not affect locals

More Terminology

- An analysis that models only a single function at a time is *intraprocedural*
- An analysis that takes multiple functions into account is *interprocedural*
- An analysis that takes the whole program into account is *whole program*
- Note: *global* analysis means “more than one basic block,” but still within a function
 - Old terminology from when computers were slow...

Data Flow Analysis and The Heap

- Data Flow is good at analyzing local variables
 - But what about values stored in the heap?
 - Not modeled in traditional data flow
- In practice: $*x := e$
 - Assume all data flow facts killed (!)
 - Or, assume write through x may affect any variable whose address has been taken
- In general, hard to analyze pointers

Proebsting's Law

- Moore's Law: Hardware advances double computing power every 18 months.
- Proebsting's Law: Compiler advances double computing power every 18 *years*.
 - Not so much bang for the buck!

DFA and Defect Detection

- LCLint - Evans et al. (UVa)
- METAL - Engler et al. (Stanford, now Coverity)
- ESP - Das et al. (MSR)
- FindBugs - Hovemeyer, Pugh (Maryland)
 - For Java. The first three are for C.
- Many other one-shot projects
 - Memory leak detection
 - Security vulnerability checking (tainting, info. leaks)