CHAPTER 19

Cascading Behavior in Networks

Diffusion in Networks

- Previously, looked at network as one aggregate population
  - Why imitating others is beneficial
    - Informational effects
    - Direct-benefit effects
- Now, look at fine structure of network as a graph, and how individuals influenced
  - Many of our interactions happen on local level

- How do new behaviors, practices, opinions, conventions, and technologies spread from person to person through a social network?
  - Informational effects (farmers adopting hybrid seed corn, doctors adopting new drugs)
  - Direct benefit effects (phone, fax, email)

- Success of innovation depends on:
  - Complexity (people can understand and implement it)
  - Observability (aware that others using it)
  - Trialability (adopt it gradually and incrementally)
  - Compatibility (with the social system it’s entering)

Modeling Diffusion Through a Network

- Diffusion from direct-benefit effects
  - As more social network neighbors adopt new behavior, benefits of you adopting it increase
  - Ex: easier to collaborate with co-workers if using compatible technologies

A Networked Coordination Game

- Each node has choice between behaviors A and B
- If nodes v and w linked by an edge, incentive for them to have their behaviors match
- Rules
  - If v and w both adopt behavior A, each get payoff of a > 0
  - If both B, each get payoff of b > 0
  - If adopt opposite behaviors, each get payoff of 0

		w	
		A	B
v	A	a, a	$0, 0$
	B	$0, 0$	b, b

Figure 19.1: *A*-*B* Coordination Game

- Some neighbors adopt A, some adopt B

- What does v do to maximize payoff?
  - Suppose p fraction of neighbors have behavior A and (1-p) fraction have behavior B
  - If v has d neighbors, pd adopt A and (1-p)d adopt B
  - If v chooses A, gets payoff pda. With B, gets payoff (1-p)db
  - A is the better choice if pda ≥ (1-p)db, AKA if p ≥ b/(a+b)

- We'll use q to describe b/(a+b)

Threshold rule: **If at least q fraction of neighbors follow behavior A, you should too**

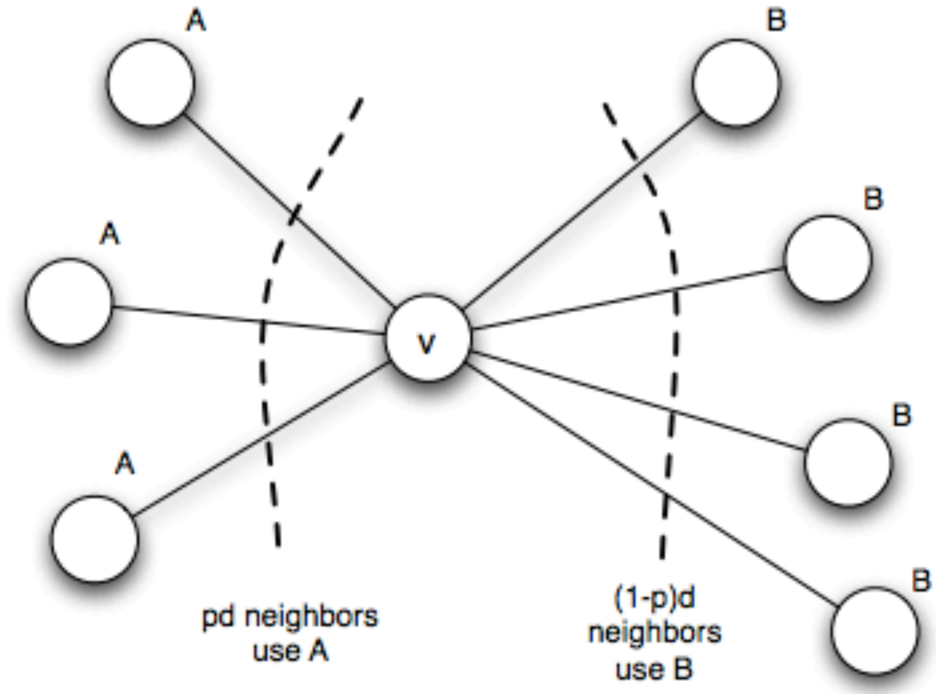


Figure 19.2: v must choose between behavior A and behavior B, based on what its neighbors are doing.

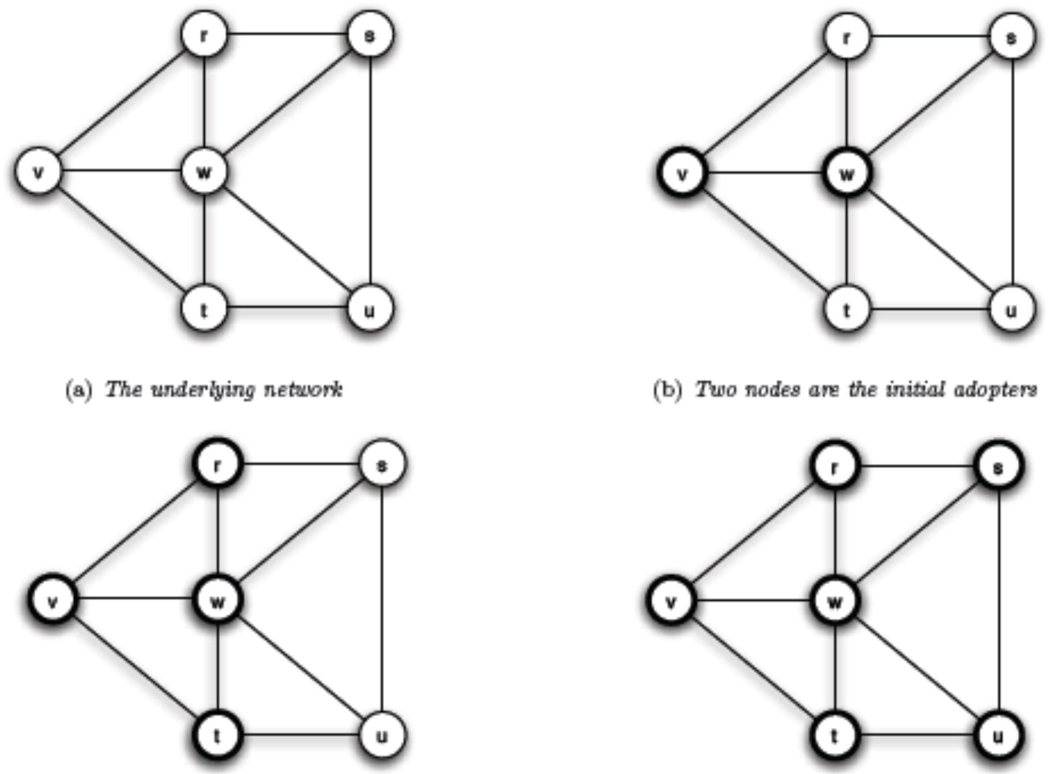
Cascading Behavior

- Suppose everyone in network initially using B as default behavior
- Small set of initial adopted decide to use A
- Some neighbors may switch to A
  - When does every node in network eventually switch to A?
  - Or what causes spread to A to stop?
- At each unit of time, each node uses threshold rule to decide whether to switch from B to A

- Suppose a=3 and b=2. With threshold formula, nodes switch from B to A if at least q=2/(3+2)=2/5 fraction of neighbors using A
- And suppose v and w are initial adopters of A, and rest use B.
- On next slide:
  - Neighbors r and t have 2/3 > 2/5 of neighbors using A, so switch to A
  - Nodes s and u only have 1/3 of neighbors using A, so don’t switch
  - At next step, s and u have 2/3 of neighbors using A, so also switch to A

- **Complete cascade** – every node in the network switches to from B to A

- Initial set of adopters causes complete cascade at threshold q



(a) *The underlying network*

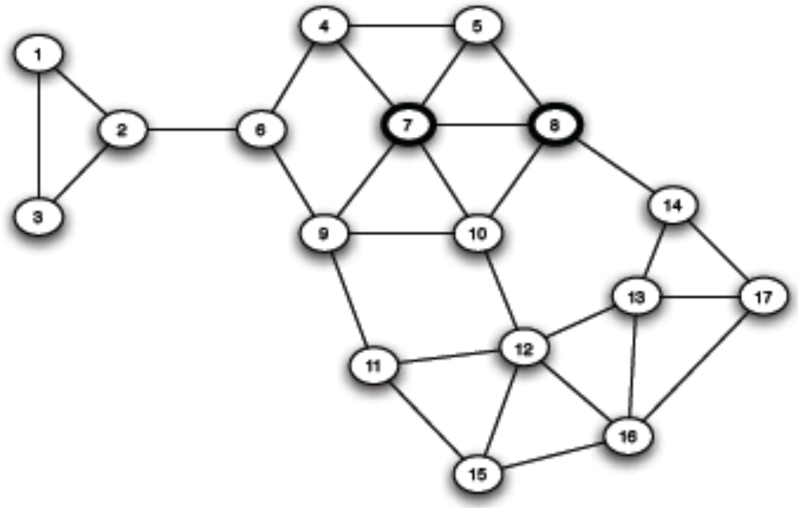
(b) *Two nodes are the initial adopters*

(c) *After one step, two more nodes have adopted*

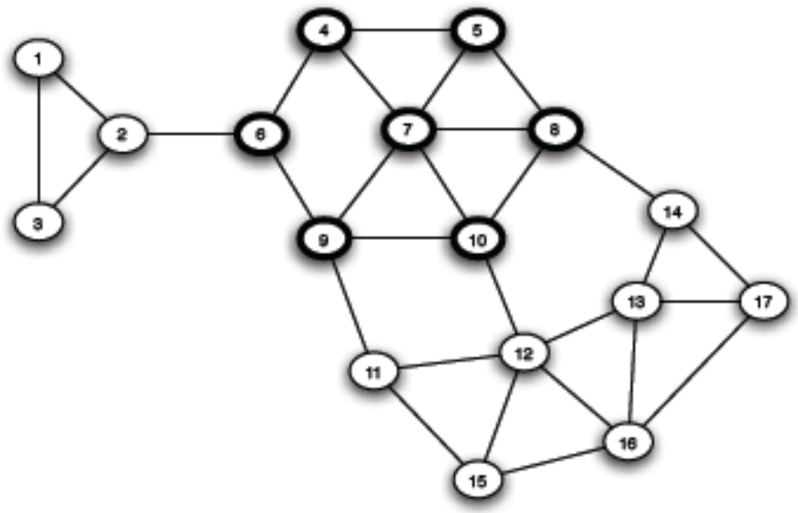
(d) *After a second step, everyone has adopted*

Figure 19.3: Starting with v and w as the initial adopters, and payoffs $a = 3$ and $b = 2$, the new behavior A spreads to all nodes in two steps. Nodes adopting A in a given step are drawn with dark borders; nodes adopting B are drawn with light borders.

- This slide shows another possibility, where adoption stops
- **Cascade** – switches run for a while but stop while nodes still using B (example in this slide)



(a) *Two nodes are the initial adopters*



(b) *The process ends after three steps*

Cascading Behavior and “Viral Marketing”

- For figure in previous slide, what if A and B were competing technologies?
- One method: A could raise **quality** of product. Could change payoff a from 3 to 4, so threshold for adopting A drops from $q=2/5$ to $q=2/(2+4)=1/3$
- If can’t change quality, **convince** key people using B to **switch to A** (like nodes 12 and 13 in previous slide, to start cascade again and eventually causing 11-17 to switch)

Cascades and Clusters

A **cluster** of density p is a set of nodes such that each node in the set has at least a p fraction of its network neighbors in the set

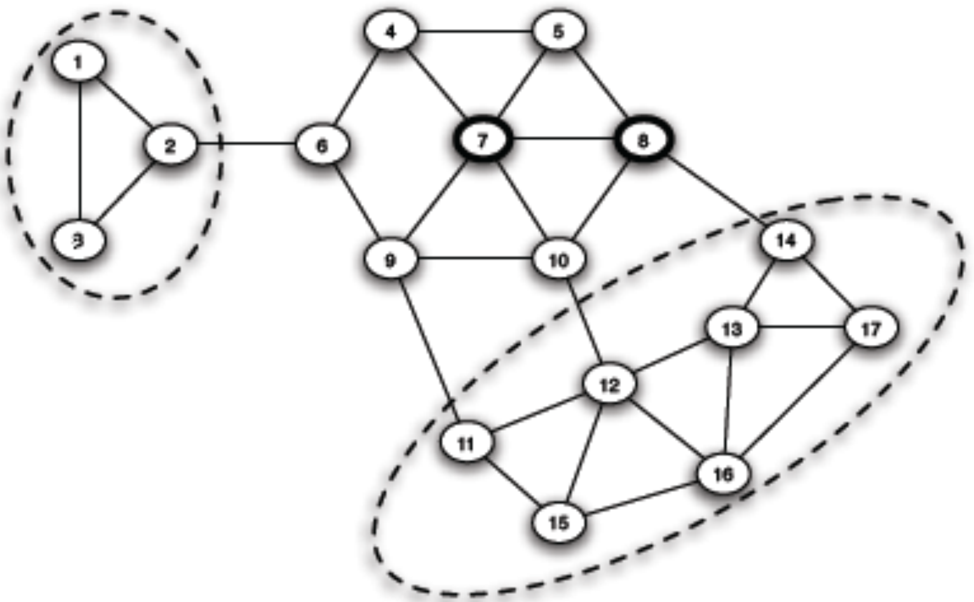


Figure 19.7: Two clusters of density 2/3 in the network from Figure 19.4.

Claim: Consider set of initial adopters of behavior A, with threshold q for nodes in remaining network to adopt A

i. If remaining network contains cluster of density greater than 1-q, set of initial adopters will not cause a complete cascade

ii. Moreover, whenever set of initial adopters doesn’t cause a complete cascade with threshold q, remaining network must contain cluster density greater than 1-q

Part (i): clusters are obstacles to cascades

- Assume opposite: node v in remaining cluster with density > 1-q adopts A at time t
- No node in its cluster adopted A before v, so at t-1, nodes using A were outside its cluster
- Since v's cluster density > 1-q, cluster outside of it has density < q. However, threshold rule requires at least q fraction of neighbors using A in order to adopt A, so **contradiction!**

Part (ii): clusters are the only obstacles to cascades

- Let S = set of nodes using B at end of process
- Claim that S is a cluster of density > 1-q
  - Consider any node w in set S
  - Since w doesn't want to switch to A, fraction of its neighbors using A must be less than q, so fraction of neighbors using B must be greater than 1-q

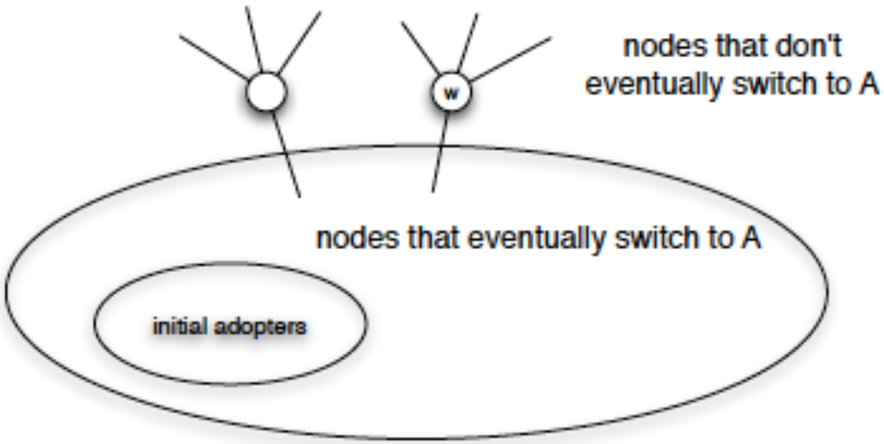


Figure 19.9: If the spread of A stops before filling out the whole network, the set of nodes that remain with B form a cluster of density greater than $1 - q$.