CHAPTER 16

Information Cascades

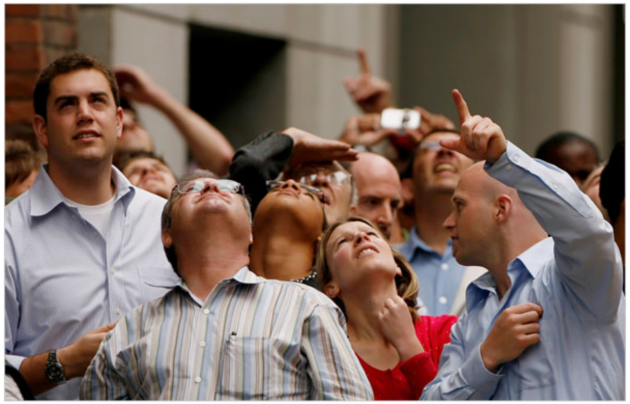
Following the Crowd

- **Information cascade** – people make decisions sequentially, with later people watching actions of earlier people and inferring what earlier people know
  - If restaurant B is almost full and next-door restaurant A is empty, the first diners to go to B conveyed information to later diners about B being better

- Drawing rational inferences from limited information

Following the Crowd

- Another example: Milgram, et al. experiment where group of people (ranging from 1 to 15 individuals) stared up in the sky on a street corner
  - With 1 person looking up, few passerbys stopped
  - With all 15 looking up, 45% passerbys stopped and looked up
  - One explanation is conformity, but another is that with more people looking up, future passerbys rationally decided that there was a good reason to look up (*information cascades*)



Informational Effects vs. Direct-Benefit Effects

- We have discussed **informational effects**, where actions of others are indirectly changing your information

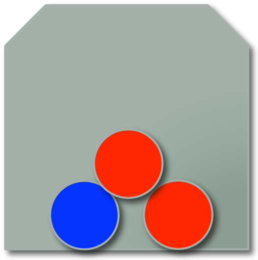
- There's also **direct-benefit effects**, where actions of others are affecting your payoffs directly
  - Example: with first fax machines, it was useless to get one if no one else owned one
  - So, important to know whether others owned a fax machine – not only because others' purchase decisions convey information, but because they *directly affect the fax machine's value to you* as a product
  - Similarly: computer operating systems, social networking sites

A Simple Herding Experiment

- Experiment by Anderson and Holt
- Situations with:
  a) Decision to be made (ex: whether to eat at a restaurant)
  b) People make the decision sequentially, and each person can observe the choices made by those who acted earlier
  c) Each person has some private information that helps guide their decision
  d) A person can't directly observe the private information that other people know, but can make an inference about this private info from what they *do*

The Experiment

- Jar at front of classroom with 3 marbles hidden
- Experimenter announces that 50% chance that jar contains 2 red, 1 blue marble (“majority-red”) and 50% chance that contains 2 blue, 1 red (“majority-blue”)
- 1 by 1, each student goes to front and draws 1 marble from the jar, looks at color without showing to rest of the class, and puts it back in the jar
- Then, student publicly announces guess that it’s majority-red or majority-blue to the class
- At end of experiment, each student who guessed correctly receives a monetary award



The Experiment

- Public announcement is the key part
  - Student who haven't gone can't see what students previously drew, but do get to hear the guesses being made

- Parallels earlier example with 2 restaurants: 1-by-1, each diner guesses which is better restaurant. Don't see reviews by earlier diners, but see which restaurant the earlier diners chose

The Experiment

- **First student:** If sees red, guesses majority-red. If sees blue, guesses majority-blue. Conveys perfect (actual) information.
- **Second student:** If sees same color as 1st student's announcement, guesses that color. If sees different color from 1st student, can guess either one – let's say she says the color she sees. Conveys perfect information.
- **Third student:**
  - If first 2 students guessed opposite colors, this student just guesses the color he sees
  - If first 2 students guessed red and he gets blue, he ignores his own private info and guesses majority-red
- *When first 2 guesses are same, 3rd student should guess this color as well, regardless of what color he draws*. Information cascade has begun.

The Experiment

- **Fourth student and onward:**
  - Consider previous 3 guessed blue
  - She knows first 2 students conveyed perfect information about what they saw
  - So she knows 3rd student would say “blue” no matter what, so his guess gives no info
  - So, 4th student in same decision-making situation as 3rd student. Sue should guess “blue” regardless of what she sees
  - All subseqent students will guess “blue” as well

The Experiment

- This shows that information cascades can lead to non-optimal outcomes
  - Suppose jar is majority-red. 1/3 chance that 1st student draws blue, and 1/3 chance that 2nd student draws blue. Since draws independent, 1/3 * 1/3 = 1/9 chance that both do
  - All subsequent guesses will be blue, so 1/9 chance of population-wide error

The Experiment

- Also shows that cascades can be fragile. Suppose first 2 guesses blue and all students say blue until students 50 and 51, who both draw red and “cheat” by showing the marbles to the rest of the class
  - Student 52 has 4 pieces of genuine info to go on – students 1, 2, 50, and 51. She now guesses based on her own draw, to break the tie
  - Easy for fresh infusion of new info to overturn initial guesses

Bayes’ Rule: A Model of Decision-Making Under Uncertainty

- Conditional probability of A given B: $$\Pr[A \mid B] = \frac{\Pr[A \cap B]}{\Pr[B]}.$$

- Conditional probability of B given A: $$\Pr[B \mid A] = \frac{\Pr[B \cap A]}{\Pr[A]} = \frac{\Pr[A \cap B]}{\Pr[A]}$$

- Rewriting these two equations gives:

$$\Pr[A \mid B] \cdot \Pr[B] = \Pr[A \cap B] = \Pr[B \mid A] \cdot \Pr[A]$$

- Dividing through by Pr[B] gives **Bayes’ Rule:**

$$\Pr[A \mid B] = \frac{\Pr[A] \cdot \Pr[B \mid A]}{\Pr[B]}.$$

Bayes' Rule Example

- Suppose 80% taxis are black, 20% are yellow
- Witness to hit-and-run states that cab involved was yellow
- Suppose that if taxi is yellow, witness will claim it's yellow 80% of time, and if taxi is black, claim it's black 80% of the time
- Let:
  - true = true color of the taxi
  - report = reported color of the cab
  - Y = yellow
  - B = black
- We're looking for Pr[true=Y | report = Y], the probability that the taxi is actually yellow, given that the witness reports that it's yellow

Bayes' Rule Example

- Using Bayes' Rule:

$$\Pr[true = Y \mid report = Y] = \frac{\Pr[true = Y] \cdot \Pr[report = Y \mid true = Y] \cdot}{\Pr[report = Y]}.$$

- We've been told that Pr[report=Y | true=Y] is 0.8 (accuracy of eyewitness testimony)
- We've also been told that Pr[true=Y] is 0.2 (frequency of yellow taxis)
- Now, we figure out Pr[report=Y] on next slide

Bayes' Rule Example

- Now, for Pr[report=Y], there are 2 ways for witness to report it's yellow: cab is actually yellow, or it's actually black
  - If actually yellow: $\Pr[true = Y] \cdot \Pr[report = Y \mid true = Y] = 0.2 \cdot 0.8 = 0.16$
  - If actually black: $\Pr[true = B] \cdot \Pr[report = Y \mid true = B] = 0.8 \cdot 0.2 = 0.16$
- Probability of report of yellow is sum of those 2 probabilities:

$$\begin{aligned}\Pr[report = Y] &= \Pr[true = Y] \cdot \Pr[report = Y \mid true = Y] + \\ &\quad \Pr[true = B] \cdot \Pr[report = Y \mid true = B] \\ &= 0.2 \cdot 0.8 + 0.8 \cdot 0.2 = 0.32.\end{aligned}$$

Bayes' Rule Example

- We put everything together in Bayes' rule:

$$
\begin{aligned}
\Pr\left[true = Y \mid report = Y\right] &= \frac{\Pr\left[true = Y\right] \cdot \Pr\left[report = Y \mid true = Y\right]}{\Pr\left[report = Y\right]} \\
&= \frac{0.2 \cdot 0.8}{0.32} \\
&= 0.5.
\end{aligned}
$$

- Conclusion: if witness says cab was yellow, it was equally likely to have been yellow or black!

Bayes' Rule in Herding Experiment

- In earlier marbles/jar experiment, each student should guess majority-blue if $\Pr[\textit{majority-blue} \mid \textit{what she has seen and heard}] > \frac{1}{2}$ and guess majority-red otherwise. If both probabilities are 0.5, then doesn't matter what she guesses

- In beginning of experiment, Pr[majority-blue] = Pr[majority-red] = ½

- Also, Pr[blue|majority-blue] = Pr[red | majority-red] = 2/3

Bayes' Rule in Herding Experiment

- Suppose first student draws blue marble and wants to determine Pr[majority-blue|blue]:

$$\Pr\left[\textit{majority-blue} \mid \textit{blue}\right] = \frac{\Pr\left[\textit{majority-blue}\right] \cdot \Pr\left[\textit{blue} \mid \textit{majority-blue}\right]}{\Pr\left[\textit{blue}\right]}.$$

- Numerator is 1/2 * 2/3 = 2/3 (reasoning from last slide)
- For denominator:

$$\begin{aligned}\Pr\left[\textit{blue}\right] &= \Pr\left[\textit{majority-blue}\right] \cdot \Pr\left[\textit{blue} \mid \textit{majority-blue}\right] + \\ &\quad \Pr\left[\textit{majority-red}\right] \cdot \Pr\left[\textit{blue} \mid \textit{majority-red}\right] \\ &= \frac{1}{2} \cdot \frac{2}{3} + \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{2}.\end{aligned}$$

- Dividing numerator/denominator: $\Pr\left[\textit{majority-blue} \mid \textit{blue}\right] = \frac{1/3}{1/2} = \frac{2}{3}.$
  - Makes sense: student should guess majority-blue when sees blue

- 2nd student similar to 1st. Let's consider 3rd student, where cascade starts to form. Suppose first 2 drew blue marbles, and 3rd student draws a red:

$$\Pr\left[\textit{majority-blue} \mid \textit{blue, blue, red}\right] = \frac{\Pr\left[\textit{majority-blue}\right] \cdot \Pr\left[\textit{blue, blue, red} \mid \textit{majority-blue}\right]}{\Pr\left[\textit{blue, blue, red}\right]}.$$

- Since draws from jar are independent:

$$\Pr\left[\textit{blue, blue, red} \mid \textit{majority-blue}\right] = \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{1}{3} = \frac{4}{27}.$$

- To calculate denominator, consider Pr[blue,blue,red] if jar is majority-blue or majority-red:

$$\begin{aligned}\Pr\left[\textit{blue, blue, red}\right] &= \Pr\left[\textit{majority-blue}\right] \cdot \Pr\left[\textit{blue, blue, red} \mid \textit{majority-blue}\right] + \\ &\quad \Pr\left[\textit{majority-red}\right] \cdot \Pr\left[\textit{blue, blue, red} \mid \textit{majority-red}\right] \\ &= \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{1}{3} + \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{2}{3} = \frac{6}{54} = \frac{1}{9}.\end{aligned}$$

Bayes' Rule in Herding Experiment

- We plug the previous slide back into Bayes' Rule:

$$\Pr\left[\textit{majority-blue} \mid \textit{blue},\ \textit{blue},\ \textit{red}\right] = \frac{\frac{4}{27} \cdot \frac{1}{2}}{\frac{1}{9}} = \frac{2}{3}.$$

- Therefore, 3rd student should guess majority-blue (and will have a 2/3 chance of being correct)
- This confirms our intuitive initial observation, that the student should ignore what he sees (red) in favor of the two guesses he's already heard (both blue)
- All future students will have same info as 3rd student, so they'll perform same calculation, resulting in an ***information cascade*** of blue guesses