CHAPTER 5

Positive and Negative Relationships

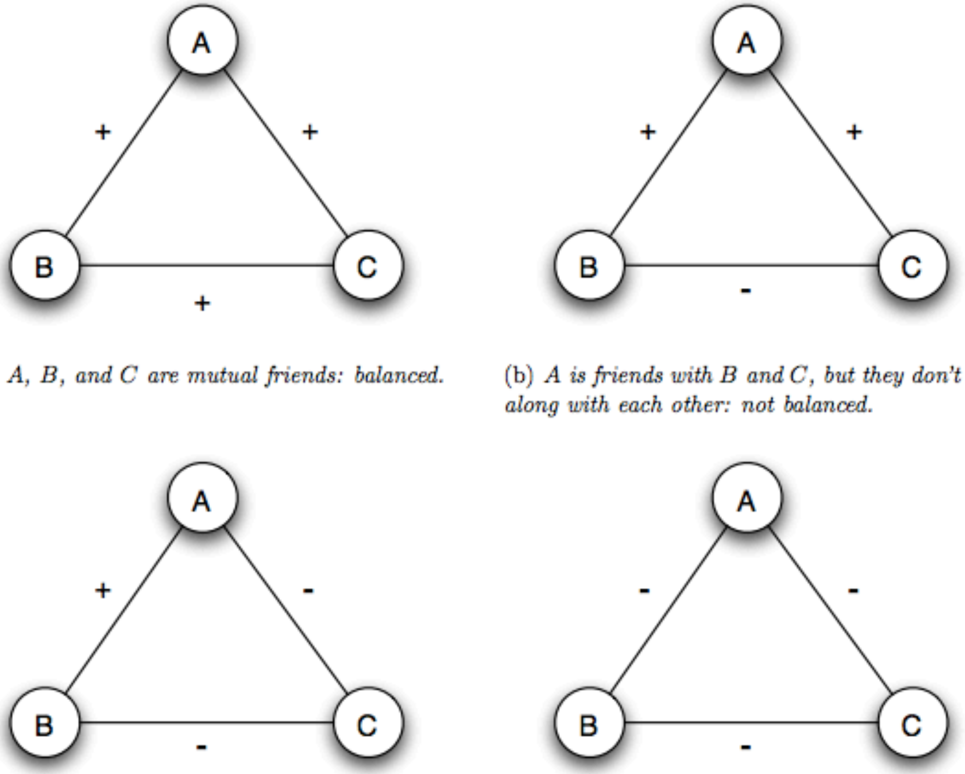
Structural Balance

- Suppose we have social network where each node joins every other node (everyone knows each other)

- Label each edge with + for friends or – for enemies

- How to label 3 edges among 3 people? Four possibilities:
  - 3 pluses is very natural: all are mutual friends.
  - 1 plus and 2 minuses is very natural: 2 are friends and have a mutual enemy
  - 2 pluses and 1 minus causes instability: A friends with B and C, but B and C are enemies. There are implicit forces on A to try to get B and C to become friends, or A to side with B or C against the other.
  - 3 minuses causes instability: Implicit forces motivating 2 of 3 to “team up” against 3rd

Triangles with 1 or 3 pluses are *balanced*, whereas 0 or 2 pluses are *unbalanced* (instability, from previous slide)



(a) *A, B, and C are mutual friends: balanced.*

(b) *A is friends with B and C, but they don't get along with each other: not balanced.*

(c) *A and B are friends with C as a mutual enemy: balanced.*

(d) *A, B, and C are mutual enemies: not balanced.*

Figure 5.1: Structural balance: Each labeled triangle must have 1 or 3 positive edges.

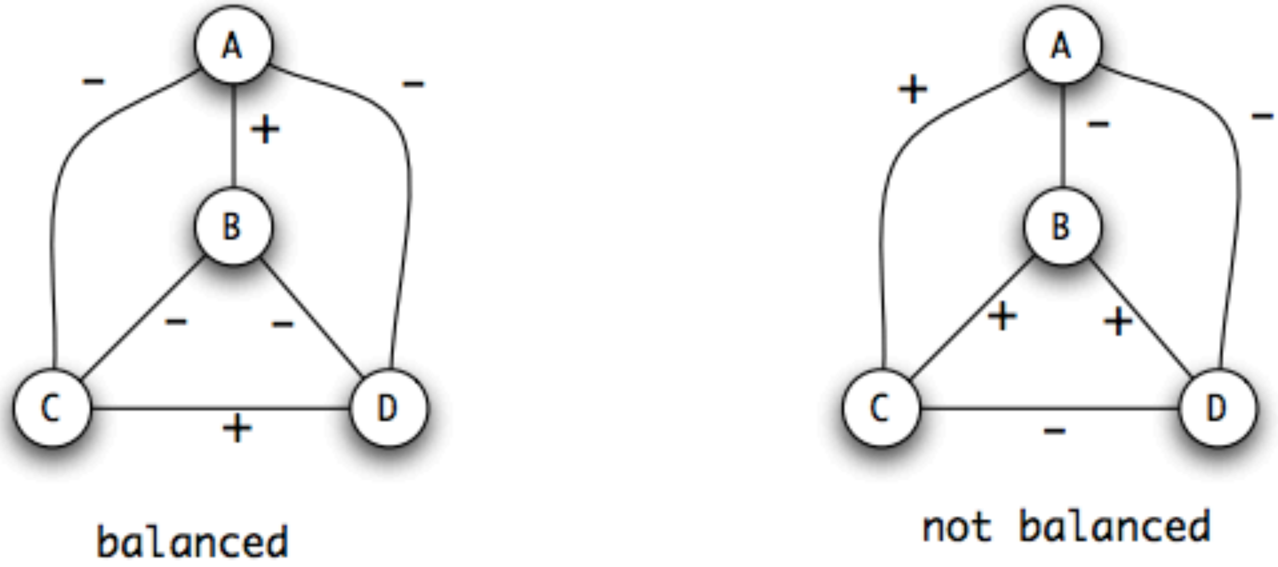


Figure 5.2: The labeled four-node complete graph on the left is balanced; the one on the right is not.

Defining Structural Balance for Networks

Structural Balance Property -

For every set of three nodes, if we consider the three edges connecting them, either all three of these edges are labeled +, or exactly one of them is labeled +.

Characterizing the Structure of Balance

- At general level, what does balanced network look like?
  - If everyone likes each other
    - All triangles have three + labels
  - 2 groups of friends with negative relations between groups

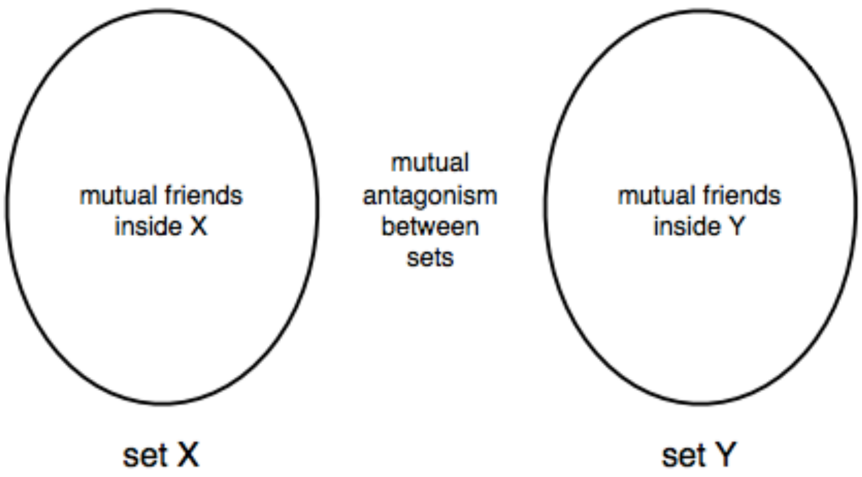


Figure 5.3: If a complete graph can be divided into two sets of mutual friends, with complete mutual antagonism between the two sets, then it is balanced. Furthermore, this is the only way for a complete graph to be balanced.

Balance Theorem

Balance Theorem – If a labeled complete graph is balanced, then either all pairs of nodes are friends, or else the nodes can be divided into two groups, X and Y, such that every pair of nodes in X likes each other, every pair of nodes in Y like each other, and everyone in X is the enemy of everyone in Y.

We showed this in the previous slide!

Proof of Balance Theorem

- Suppose we have a *balanced, labeled* complete graph. Must show that has structure in the claim.
  - If no negative edges, then everyone is friends, and proof done
  - Otherwise, at least 1 negative edge
- Pick some node A from network. Divide into X (A and all of A's friends) and Y (all of A's enemies)

- To satisfy claim, this must be true:

  i) Every 2 nodes in X are friends

  ii) Every 2 nodes in Y are friends

  iii) Every node in X is enemy of every node in Y

  Proof of these on next slide!

i) Every 2 nodes in X are friends

- Already know A friends with all nodes in X
- If A's 2 friends (B and C) they were enemies, would form 2 pluses and 1 minus, a violation of balance condition
- We know network is balanced, so B and C must be friends

ii) Every 2 nodes in Y are friends

- A enemies with D and E in Y
- If D and E enemies, then no pluses, a violation of balance condition

iii) Every node in X enemy of every node in Y

- Consider node B in X and node D in Y
- A friends with B and enemies with D.
- If B and D friends, then A, B, D form 2 pluses, violation of balance

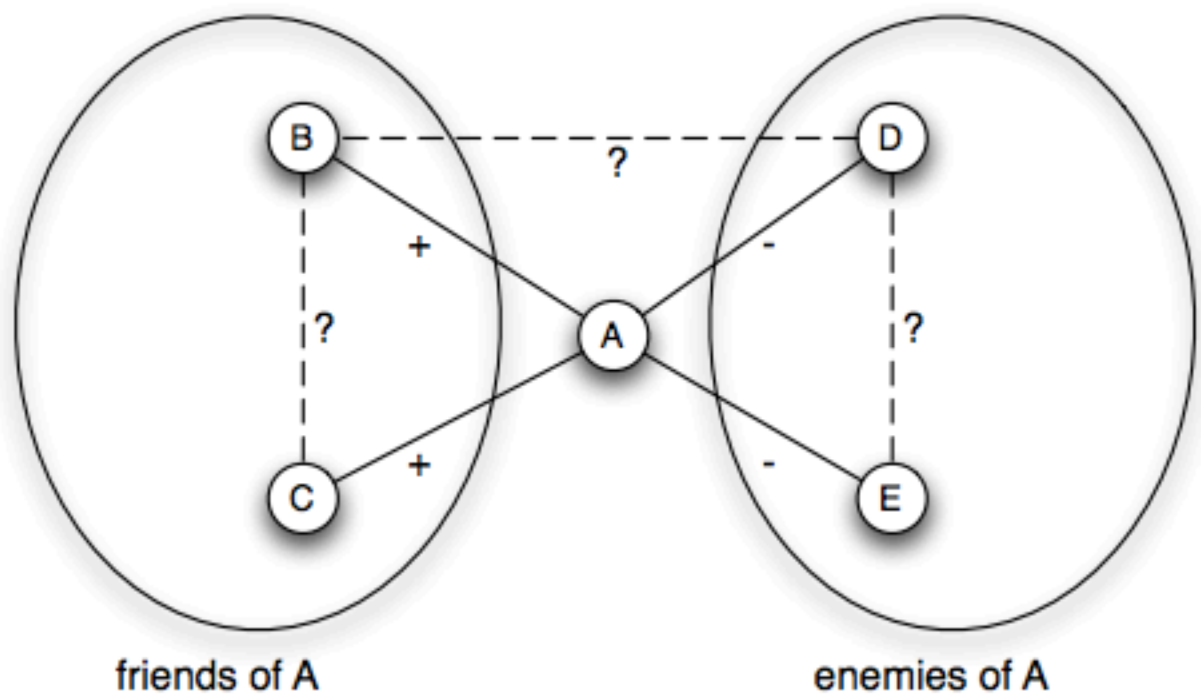


Figure 5.4: A schematic illustration of our analysis of balanced networks. (There may be other nodes not illustrated here.)