Bottom-up Parsing

Roadmap (Where are we?)

We set out to study parsing

- Predictive top-down parsing
  - → FIRST, FOLLOW, FIRST+
  - → The LL(1) condition
  - → Table-driven LL(1) parsers
- Bottom-up parsing ←
  - → Finding reductions
  - → Shift-reduce parsers

Parsing Techniques: Bottom-up parsers

LR(1), operator precedence

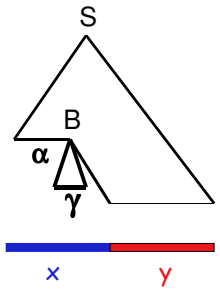
1 input symbol lookahead

construct rightmost derivation (backwards)

input: read left-to-right

rule B ::= γ

$S \Rightarrow^*_{rm} \alpha B y \Rightarrow_{rm} \alpha \gamma y \Rightarrow^*_{rm} x y$



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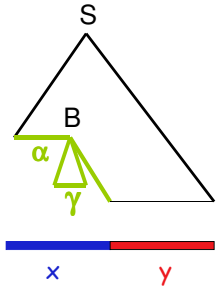
upper fringe



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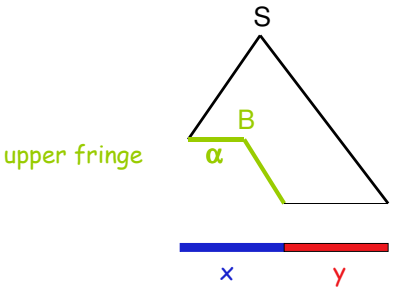

Bottom-up Parsing

A bottom-up parser builds a derivation by working from the input sentence back toward the start symbol *S*

$$S \Rightarrow \gamma_0 \Rightarrow \gamma_1 \Rightarrow \gamma_2 \Rightarrow \ldots \Rightarrow \gamma_{n-1} \Rightarrow \gamma_n \Rightarrow \textit{sentence}$$

bottom-up

To reduce γ_i to γ_{i-1} match some *rhs* β against γ_i then replace β with its corresponding *lhs*, *A*. *(assuming the production $A \rightarrow \beta$)*

In terms of the parse tree, this is working from leaves to root

- Nodes with no parent in a partial tree form its *upper fringe*
- Since each replacement of β with *A* shrinks the upper fringe, we call it a *reduction*.

The parse tree need not be built, it can be simulated

$$|\textit{parse tree nodes}| = |\textit{words}| + |\textit{reductions}|$$

Finding Reductions

Consider the simple grammar

1	*Goal*	→	a *A B* e
2	*A*	→	*A* b c
3		|	b
4	*B*	→	d

And the input string abbcde

Sentential Form	*Next Reduction* *Prod'n*	*Pos'n*
abbcde	3	2
a *A* bcde	2	4
a *A* de	4	3
a *A B* e	1	4
Goal	—	—

The trick is scanning the input and finding the next reduction
The mechanism for doing this must be efficient

Finding Reductions (Handles)

The parser must find a substring β of the tree's frontier that
matches some production $A \rightarrow \beta$ that occurs as one step in the rightmost derivation

Informally, we call this substring β a *handle*

Formally,

A *handle* of a right-sentential form γ is a pair $\langle A \rightarrow \beta, k \rangle$ where $A \rightarrow \beta \in P$ and k is the position in γ of β's rightmost symbol.

If $\langle A \rightarrow \beta, k \rangle$ is a handle, then replacing β at k with A produces the right sentential form from which γ is derived in the rightmost derivation.

Because γ is a right-sentential form, the substring to the right of a handle contains only terminal symbols

$\Rightarrow$ the parser doesn't need to scan past the handle (only lookahead)

Example (a very busy slide)

1	*Goal*	→	*Expr*
2	*Expr*	→	*Expr* + *Term*
3		|	*Expr* - *Term*
4		|	*Term*
5	*Term*	→	*Term* * *Factor*
6		|	*Term* / *Factor*
7		|	*Factor*
8	*Factor*	→	number
9		|	id
10		|	(*Expr*)

Prod'n.	*Sentential Form*	*Handle*
—	*Goal*	—
1	*Expr*	1,1
3	*Expr* - *Term*	3,3
5	*Expr* - *Term* * *Factor*	5,5
9	*Expr* - *Term* * <id,y>	9,5
7	*Expr* - *Factor* * <id,y>	7,3
8	*Expr* - <num,2> * <id,y>	8,3
4	*Term* - <num,2> * <id,y>	4,1
7	*Factor* - <num,2> * <id,y>	7,1
9	<id,x> - <num,2> * <id,y>	9,1

The expression grammar *Handles for rightmost derivation of* x - 2 * y

This is the inverse of Figure 3.9 in EaC

Finding Reductions (Handles)

Critical Insight *(Theorem)*

If G is unambiguous, then every right-sentential form has a unique handle.

If we can find those handles, we can build a derivation !

Sketch of Proof:

1. *G* is unambiguous ⇒ rightmost derivation is unique
2. ⇒ a unique production $A \rightarrow \beta$ applied to derive γ_i from γ_{i-1}
3. ⇒ a unique position ***k*** at which $A \rightarrow \beta$ is applied
4. ⇒ a unique handle $\langle A \rightarrow \beta, k \rangle$

This all follows from the definitions

Handle-pruning, Bottom-up Parsers

The process of discovering a handle & reducing it to the appropriate left-hand side is called *handle pruning*

Handle pruning forms the basis for a bottom-up parsing method

To construct a rightmost derivation

$S \Rightarrow \gamma_0 \Rightarrow \gamma_1 \Rightarrow \gamma_2 \Rightarrow \ldots \Rightarrow \gamma_{n-1} \Rightarrow \gamma_n \Rightarrow w$

Apply the following simple algorithm

for i ← n to 1 by -1
Find the handle $\langle A_i \rightarrow \beta_i, k_i \rangle$ in γ_i
Replace β_i *with* A_i *to generate* γ_{i-1}

This takes *2n* steps

Handle-pruning, Bottom-up Parsers

One implementation technique is the *shift-reduce parser*

```
push INVALID
token ← next_token( )
repeat until (top of stack = Goal and token = EOF)
    if the top of the stack is a handle A→β
        then    // reduce β to A
            pop |β| symbols off the stack
            push A onto the stack
        else if (token ≠ EOF)
            then // shift
                push token
                token ← next_token( )
            else    // need to shift, but out of input
                report an error
```

How do errors show up?

- **failure to find a handle**
- **hitting EOF & needing to shift (final else clause)**

Either generates an error

Figure 3.11 in EAC

Back to x - 2 * y

Stack	Input	Handle	Action
$	id – num * id	*none*	shift
$ id	– num * id		

1. Shift until the top of the stack is the right end of a handle
2. Find the left end of the handle & reduce

Back to x - 2 * y

Stack	Input	Handle	Action
$	id – num * id	*none*	shift
$ id	– num * id	9,1	red. 9
$ *Factor*	– num * id	7,1	red. 7
$ *Term*	– num * id	4,1	red. 4
$ *Expr*	– num * id		

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$ *Term*	– num * id	4,1	red. 4
$ *Expr*	– num * id	*none*	shift
$ *Expr* –	num * id	*none*	shift
$ *Expr* – num	* id		

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$ *Expr*	– num * id	*none*	shift
$ *Expr* –	num * id	*none*	shift
$ *Expr* – num	* id	8,3	red. 8
$ *Expr* – *Factor*	* id	7,3	red. 7
$ *Expr* – *Term*	* id		

1. Shift until the top of the stack is the right end of a handle
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$ *Expr* – *Term*	* id	*none*	shift
$ *Expr* – *Term* *	id	*none*	shift
$ *Expr* – *Term* * id			

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$ *Expr* – *Term*	* id	*none*	shift
$ *Expr* – *Term* *	id	*none*	shift
$ *Expr* – *Term* * id		9,5	red. 9
$ *Expr* – *Term* * *Factor*		5,5	red. 5
$ *Expr* – *Term*		3,3	red. 3
$ *Expr*		1,1	red. 1
$ *Goal*		*none*	accept

5 shifts +
9 reduces +
1 accept

1. Shift until the top of the stack is the right end of a handle
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Back to x - 2 * y

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$ *Expr* – *Term*		3,3	red. 3
$ *Expr*		1,1	red. 1
$ *Goal*		*none*	accept

shift here

reduce here

1. Shift until the top of the stack is the right end of a handle
2. Find the left end of the handle & reduce

Back to x - 2 * y

Stack	Input	Action
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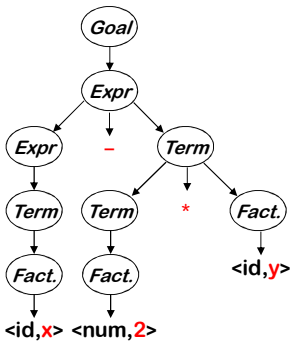
$ *Expr* – num	* id	red. 8
$ *Expr* – *Factor*	* id	red. 7
$ *Expr* – *Term*	* id	shift
$ *Expr* – *Term* *	id	shift
$ *Expr* – *Term* * id		red. 9
$ *Expr* – *Term* * *Factor*		red. 5
$ *Expr* – *Term*		red. 3
$ *Expr*		red. 1
$ *Goal*		accept



Shift-reduce Parsing

Shift reduce parsers are easily built and easily understood

A shift-reduce parser has just four actions

- *Shift* — next word is shifted onto the stack
- *Reduce* — right end of handle is at top of stack
  - Locate left end of handle within the stack
  - Pop handle off stack & push appropriate *lhs*
- *Accept* — stop parsing & report success
- *Error* — call an error reporting/recovery routine

Accept & Error are simple
Shift is just a push and a call to the scanner
Reduce takes |*rhs*| pops & 1 push
If handle-finding requires state, put it in the stack ⇒ *2x work*

Handle finding is key
- handle is on stack
- finite set of handles

⇒ use a DFA !

An Important Lesson about Handles

To be a handle, a substring of a sentential form γ must have two properties:

- → It must match the right hand side β of some rule $A \rightarrow \beta$
- → There must be some rightmost derivation from the goal symbol that produces the sentential form γ with $A \rightarrow \beta$ as the last production applied

- Simply looking for right hand sides that match strings is not good enough
- Critical Question: How can we know when we have found a handle without generating lots of different derivations?
  - → Answer:
    LR(1) parsers build a DFA that runs over the stack & finds them. One token look-ahead may be used to determine the next action (shift or reduce) in each state of the DFA.
    LR(0) parsers: no look-ahead.

LR(1) Parsers

- LR(1) parsers are table-driven, shift-reduce parsers that use a limited right context (1 token) for handle recognition
- LR(1) parsers recognize languages that have an LR(1) grammar

Informal definition:

A grammar is LR(1) if, given a rightmost derivation

$$S \Rightarrow \gamma_0 \Rightarrow \gamma_1 \Rightarrow \gamma_2 \Rightarrow \ldots \Rightarrow \gamma_{n-1} \Rightarrow \gamma_n \Rightarrow \textit{sentence}$$

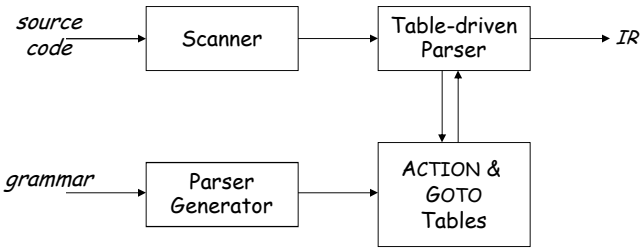
We can

1. *isolate the handle of each right-sentential form* γ_i, and
2. *determine the production by which to reduce,*

by scanning γ_i from left-to-right, going at most 1 symbol beyond the right end of the handle of γ_i

LR(1) Parsers

A table-driven LR(1) parser looks like



Tables *can* be built by hand

However, this is a perfect task to automate

LR(1) Skeleton Parser

```
stack.push(INVALID); stack.push(s0);
not_found = true;
token = scanner.next_token();
do while (not_found) {
        s = stack.top();
        if ( ACTION[s,token] == "reduce A→β" ) then {
                stack.popnum(2*|β|); // pop 2*|β| symbols
        s = stack.top();
        stack.push(A);
        stack.push(GOTO[s,A]);
        }
        else if ( ACTION[s,token] == "shift si" ) then {
                stack.push(token); stack.push(si);
                token ← scanner.next_token();
        }
        else if ( ACTION[s,token] == "accept"
                                & token == EOF )
                then not_found = false;
        else report a syntax error and recover;
}
report success;
```

The skeleton parser

- uses ACTION & GOTO tables
- does |*words*| shifts
- does |derivation| reductions
- does 1 accept
- detects errors by failure of 3 other cases