

Instruction Scheduling

What Makes Code Run Fast?

- Many operations have non-zero latencies
- Modern machines can issue several operations per cycle
- Execution time is **order-dependent** (and has been since the 60's)

Assumed latencies (conservative)

Operation	Cycles
load	3
store	3
loadl	1
add	1
mult	2
fadd	1
fmult	2
shift	1
branch	0 to 8

- Loads & stores may or may not block
 - > Non-blocking $\Rightarrow$ fill those issue slots
- Branch costs vary with path taken
- Branches typically have delay slots
 - > Fill slots with unrelated operations
 - > Percolates branch upward
- Scheduler should hide the latencies

Example

$$w \leftarrow w * 2 * x * y * z$$

Simple schedule

```
1 loadAl    r0,@w  => r1
4 add       r1,r1  => r1
5 loadAl    r0,@x  => r2
8 mult      r1,r2  => r1
9 loadAl    r0,@y  => r2
12 mult     r1,r2  => r1
13 loadAl    r0,@z  => r2
16 mult     r1,r2  => r1
18 storeAl   r1     => r0,@w
21 r1 is free
```

2 registers, 20 cycles

Schedule loads early

```
1 loadAl    r0,@w  => r1
2 loadAl    r0,@x  => r2
3 loadAl    r0,@y  => r3
4 add       r1,r1  => r1
5 mult      r1,r2  => r1
6 loadAl    r0,@z  => r2
7 mult      r1,r3  => r1
9 mult      r1,r2  => r1
11 storeAl   r1     => r0,@w
14 r1 is free
```

3 registers, 13 cycles

Reordering operations for speed is called instruction scheduling

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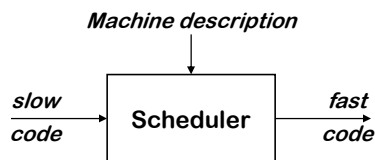
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Instruction Scheduling (Engineer's View)

The Problem

Given a code fragment for some target machine and the latencies for each individual operation, reorder the operations to minimize execution time

The Concept



The task

- Produce correct code
- Minimize wasted cycles
- Avoid spilling registers
- Operate efficiently

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Instruction Scheduling (The Abstract View)

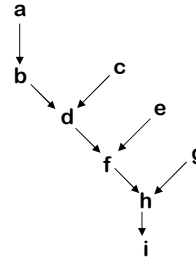
To capture properties of the code, build a **precedence graph** G

- Nodes $n \in G$ are operations with $type(n)$ and $delay(n)$
- An edge $e = (n_1, n_2) \in G$ if & only if n_2 uses the result of n_1

```

a: loadAl    r0,@w  => r1
b: add      r1,r1   => r1
c: loadAl    r0,@x  => r2
d: mult     r1,r2   => r1
e: loadAl    r0,@y  => r2
f: mult     r1,r2   => r1
g: loadAl    r0,@z  => r2
h: mult     r1,r2   => r1
i: storeAl   r1      => r0,@w
    
```

The Code



The Precedence Graph

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Instruction Scheduling (Definitions)

A **correct schedule** S maps each $n \in N$ into a non-negative integer representing its cycle number, and

1. $S(n) \geq 0$, for all $n \in N$, obviously
2. If $(n_1, n_2) \in E$, $S(n_1) + delay(n_1) \leq S(n_2)$
3. For each type t , there are no more operations of type t in any cycle than the target machine can issue

The **length** of a schedule S , denoted $L(S)$, is

$$L(S) = \max_{n \in N} (S(n) + delay(n))$$

The goal is to find the shortest possible correct schedule.

S is **time-optimal** if $L(S) \leq L(S_i)$, for all other schedules S_i

A schedule might also be optimal in terms of registers, power, or space....

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Instruction Scheduling (What's so difficult?)

Critical Points

- All operands must be available
- Multiple operations can be **ready**
- Moving operations can lengthen register lifetimes
- Placing uses near definitions can shorten register lifetimes
- Operands can have multiple predecessors

Together, these issues make scheduling **hard** (NP-Complete)

Local scheduling is the simple case

- Restricted to straight-line code
- Consistent and predictable latencies

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Instruction Scheduling

The big picture

1. Build a precedence graph, P
2. Compute a **priority function** over the nodes in P
3. Use list scheduling to construct a schedule, one cycle at a time
 - a. Use a queue of operations that are ready
 - b. At each cycle
 - I. Choose a ready operation and schedule it
 - II. Update the ready queue

Local list scheduling

- The dominant algorithm for twenty years
- A greedy, heuristic, local technique

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Local List Scheduling

```

Cycle ← 1
Ready ← leaves of P
Active ← ∅

while (Ready ∪ Active ≠ ∅)
  if (Ready ≠ ∅) then
    ← Removal in priority order
    remove an op from Ready
    S(op) ← Cycle
    Active ← Active ∪ op
    ← op has completed execution
    Cycle ← Cycle + 1
    for each op ∈ Active
      ← If successor's operands are ready, put it on Ready
      if (S(op) + delay(op) ≤ Cycle) then
        remove op from Active
        for each successor s of op in P
          if (s is ready) then
            Ready ← Ready ∪ s
  
```

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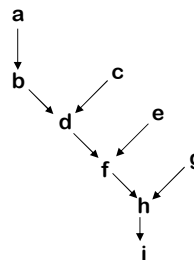
Scheduling Example

1. Build the precedence graph

```

a: loadAl    r0,@w  ⇒ r1
b: add       r1,r1   ⇒ r1
c: loadAl    r0,@x  ⇒ r2
d: mult      r1,r2   ⇒ r1
e: loadAl    r0,@y  ⇒ r2
f: mult      r1,r2   ⇒ r1
g: loadAl    r0,@z  ⇒ r2
h: mult      r1,r2   ⇒ r1
i: storeAl   r1      ⇒ r0,@w
  
```

The Code



The Precedence Graph

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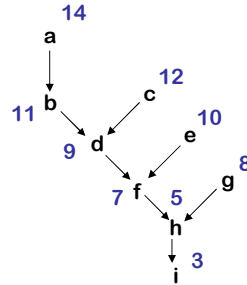
Scheduling Example

1. Build the precedence graph
2. Determine priorities: longest latency-weighted path

```

a: loadAl    r0,@w  => r1
b: add       r1,r1  => r1
c: loadAl    r0,@x  => r2
d: mult      r1,r2  => r1
e: loadAl    r0,@y  => r2
f: mult      r1,r2  => r1
g: loadAl    r0,@z  => r2
h: mult      r1,r2  => r1
i: storeAl   r1     => r0,@w
    
```

The Code



The Precedence Graph

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Scheduling Example

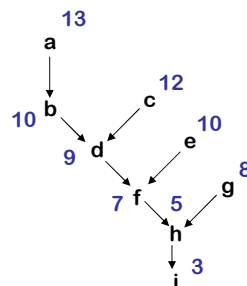
1. Build the precedence graph
2. Determine priorities: longest latency-weighted path
3. Perform list scheduling

```

1) a: loadAl    r0,@w  => r1
2) c: loadAl    r0,@x  => r2
3) e: loadAl    r0,@y  => r3
4) b: add       r1,r1  => r1
5) d: mult      r1,r2  => r1
6) g: loadAl    r0,@z  => r2
7) f: mult      r1,r3  => r1
9) h: mult      r1,r2  => r1
11) i: storeAl   r1     => r0,@w
    
```

The Code

New register name used



The Precedence Graph

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Detailed Scheduling Algorithm I

Idea: Keep a collection of worklists $W[c]$, one per cycle

→ We need $MaxC = \max \text{delay} + 1$ such worklists

Code:

```

for each  $n \in N$  do begin  $\text{count}[n] := 0$ ;  $\text{earliest}[n] = 0$  end
for each  $(n1, n2) \in E$  do begin
     $\text{count}[n2] := \text{count}[n2] + 1$ ;
     $\text{successors}[n1] := \text{successors}[n1] \cup \{n2\}$ ;
end
for  $i := 0$  to  $MaxC - 1$  do  $W[i] := \emptyset$ ;
 $Wcount := 0$ ;
for each  $n \in N$  do
    if  $\text{count}[n] = 0$  then begin
         $W[0] := W[0] \cup \{n\}$ ;  $Wcount := Wcount + 1$ ;
    end
 $c := 0$ ; //  $c$  is the cycle number
 $cW := 0$ ; //  $cW$  is the number of the worklist for cycle  $c$ 
 $\text{instr}[c] := \emptyset$ ;

```

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Detailed Scheduling Algorithm II

```

while  $Wcount > 0$  do begin
    while  $W[cW] = \emptyset$  do begin
         $c := c + 1$ ;  $\text{instr}[c] := \emptyset$ ;  $cW := \text{mod}(cW + 1, MaxC)$ ;
    end
     $\text{nextc} := \text{mod}(c + 1, MaxC)$ ;
    while  $W[cW] \neq \emptyset$  do begin
        Priority → select and remove an arbitrary instruction  $x$  from  $W[cW]$ ;
        if  $\exists$  free issue units of type( $x$ ) on cycle  $c$  then begin
             $\text{instr}[c] := \text{instr}[c] \cup \{x\}$ ;  $Wcount := Wcount - 1$ ;
            for each  $y \in \text{successors}[x]$  do begin
                 $\text{count}[y] := \text{count}[y] - 1$ ;
                 $\text{earliest}[y] := \max(\text{earliest}[y], c + \text{delay}(x))$ ;
                if  $\text{count}[y] = 0$  then begin
                     $\text{loc} := \text{mod}(\text{earliest}[y], MaxC)$ ;
                     $W[\text{loc}] := W[\text{loc}] \cup \{y\}$ ;  $Wcount := Wcount + 1$ ;
                end
            end
        else  $W[\text{nextc}] := W[\text{nextc}] \cup \{x\}$ ;
    end
end

```

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More List Scheduling

List scheduling breaks down into two distinct classes

Forward list scheduling

- Start with available operations
- Work forward in time
- Ready $\Rightarrow$ all operands available

Backward list scheduling

- Start with no successors
- Work backward in time
- Ready $\Rightarrow$ latency covers uses

Variations on list scheduling

- Prioritize critical path(s)
- Schedule last use as soon as possible
- Depth first in precedence graph (minimize registers)
- Breadth first in precedence graph (minimize interlocks)
- Prefer operation with most successors

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