Announcements

• Maryland Women’s basketball team clinched the Big 10 title!

• I will post examples from class online starting today. **Passwords given out during class only!**

• Homework #3 is due tomorrow.

• Midterm #1 is on March 5\textsuperscript{th} (9 days from now.)
Proof by Contradiction

Sometimes easier than proving something directly:

**Claim:** P.

**Proof:**

Assume ~P.

...

[Contradiction].

Therefore, P.
Proofs by Contradiction

Lemma 1: \((\forall n \in \mathbb{N})[ n^2 \text{ even } \iff n \text{ is even}]\)
(We already proved this! Really??)

- Claim: \((\forall x, y \in \mathbb{Z})[x^2 - 4y \neq 2]\)
A Famous Proof by Contradiction

Theorem: $\sqrt{2}$ is irrational.
Proven around 500 BC, probably by Hippasus
We’ll need these...

Lemma 2: \((\forall x, y \in \mathbb{N}^>1) \text{ if } x \mid y \text{ then } x \nmid (y + 1)\)

(Let’s prove this...)

Lemma 3: Every natural number (greater than 1) has a prime factor.

(We’ll prove this later...)
Another Famous Proof by Contradiction

Theorem: There are infinitely many primes.
Proven around 300 BC, by Euclid
How Smart is a Computer?

• Can software be written to do certain kinds of proofs?

• Can there be a TruthMaster® program that can decide whether or not statements of predicate logic are true (in every interpretation)?

• How good can a program be at analyzing source code of other programs?
The Halting Problem

Question: Is it possible to write a computer program called **CodeAnalyzer®** with the following characteristics?

- The CodeAnalyzer program takes two inputs:
  1. Source code of some computer program, P
  2. Data (D) that could be used as input for the program P
- CodeAnalyzer will tell us whether or not the program P would eventually *halt* (when run with input D), by returning either “IT WOULD HALT” or “IT WOULD RUN FOREVER”
Yet Another Famous Proof by Contradiction

Theorem: The “CodeAnalyzer” program cannot exist.
Proven by Alan Turing in 1936. (He was 24 years old.)
Similar Results

**Undecidable** questions about “What happens when Program P is run on input D”:

- Will it halt? (Halting Problem).
- Will it ever reach line 679?
- Will the output include the string “CMSC 250 is fun”? 
- Will there be any output?
- Etc.
Math Humor (by Contradiction)

• Claim: All natural numbers are interesting.
Unit 5
Focus on Number Theory
Fundamental Theorem of Arithmetic  
(Unique Prime Factorization Theorem)

Theorem: For any $n \in \mathbb{N}$, $n$ can be expressed as the product of primes in a unique way.

Examples.

In proofs, we will write:

$$ n = p_1^{e_1} p_2^{e_2} p_3^{e_3} \ldots p_k^{e_k} $$