Announcements

- Homework #7 has been posted.
- Monday 4/13 is the last day to drop with a ‘W’
- 2nd midterm is in two weeks (4/23)

Recall: Powerset

$\mathcal{P}$(A) is the set of all subsets of A

- Example
- What does an element of the powerset look like?

Proofs about power sets

- Claim: $A \subseteq B \rightarrow \mathcal{P}(A) \subseteq \mathcal{P}(B)$

- Claim: For finite sets, A: $[n(A) = k \rightarrow n(\mathcal{P}(A)) = 2^k]$

  [Think about inductive step with a small example.]

- Claim: $\mathcal{P}(A) \cap \mathcal{P}(B) = \mathcal{P}(A \cap B)$

  Does this work for union?

Partitions of a set

- A collection of nonempty sets $\{A_1, A_2, \ldots, A_n\}$ is a *partition* of the set A if and only if

  1. $A = A_1 \cup A_2 \cup \ldots \cup A_n$
  2. $A_1, A_2, \ldots, A_n$ are mutually disjoint

- An infinite set can be partitioned. The partitions can be infinite, or can be finite.

- Examples

Rusell's Paradox

- Consider the set S = {A | A is a set and A $\notin$ A}

  Is S an element of itself?

Unit 9
Counting and Probability

Wait... don’t we already know how to count?

Probability

- What is probability?
- Intuitive examples
  - What is the smallest probability possible?
  - What is the largest probability possible?
- Sometimes expressed as fractions, sometimes decimals, sometimes as a percentage, sometimes (for small values) “1 in x”.

Probability Definitions

- Sample space = set of all possible outcomes
- Event = any subset of the sample space
- Convenient examples (coin flips, cards, dice, sports, etc.)

Classical Probability Formula

If we have a Sample Space, S, **where every element of the sample space is equally** likely then the probability of an event, E is given by:

$$P(E) = \frac{n(E)}{n(S)}$$

- Examples (coins flips, cards)
- Examples where the formula must be used carefully:
  - A couple will have two children. What is the probability that they will both be girls? (Assuming boys and girls are equally likely)
  - Roll two dice. What is the probability of rolling a total of 12?

Tasks with Multiple "Steps"

- Example: Eating breakfast

  Your breakfast will be cereal and a beverage.

  – The cereal can be any of: Rice Krispies, cornflakes, Raisin Bran, or Cheerios.

  – The beverage can be any of: coffee, orange juice, or milk.

  How many ways can you have breakfast?

How can we extend this to a dinner where we choose an appetizer, a main course, a side dish, a desert, and a beverage?

The Multiplication Rule

How many ways are there to accomplish a Task that consists of k steps in which we have choices for how to perform each step?

$$n_1 * n_2 * \cdots * n_k$$

(where n_i = the number of ways to perform step #i)

Using the multiplication rule for selecting a PIN

Assume that you will be assigned a **random** 4-digit PIN number, like: 7097

- How many 4-digit PINs are possible?

- What is the probability that your PIN will contain only even digits?
- What is the probability that all of the digits in your PIN are unique?
- What is the probability that your PIN will never have the same digit twice in a row?

Using the multiplication rule for License Plates

Maryland license plates consist of a digit, two letters, and then 4 digits:

5CM8291

- How many Maryland license plates are possible?

- Suppose the license plates were assigned randomly (they are not, just pretend). What would be the probability of getting a plate where all of the digits are the same and both of the letters are the same? (For example: 3BB3333).