Announcements

- Homework #9 is due tomorrow. This is the last homework.
- We will have one more quiz tomorrow.
- The final exam is on 5/16 (Saturday) at 4:00PM.
Birthday Question (Revisited…)  

How many people do you need to have in a room so that it is more than 50% likely that some pair of people in the room have the same birthday?

- Solution #1 (an approximation that is easy to compute)
- Solution #2 (an exact answer that is harder to compute)

So let’s write a program to do it!

- Can we compute numerator first, then denominator and then divide?
- How would you write the Java method:

```java
static long choose(int n, int r)
```
Unit 11
Relations
Relations

A relation (among sets) is a subset of their Cartesian product.

Relations can involve any number of sets, but frequently they are binary (two sets).
Examples of Binary Relations

Let $S = \{\text{Students at Maryland}\}$
Let $F = \{\text{faculty members at Maryland}\}$

Define relation $R$ on $S \times F$ by:

$R = \{ <x, y> \in S \times F : x \text{ has been in a class taught by } y \}$

Notation:

$aRb$ means $<a, b> \in R$
Examples of Binary Relations

- Any predicate with two free variables (over fixed domains) defines a binary relation over the same domains:
  For all $x \in A$, $y \in B$
  \[ xRy \iff P(x, y) \text{ is true} \]

- $<$ is a binary relation on $\mathbb{R}$, $\mathbb{R}$, or $\mathbb{Z}$, $\mathbb{Z}$, etc.

- Any function can be thought of as a binary relation
  (Can any binary relation be thought of as a function?)

- $=$ can be thought of as a (simple) binary relation
Ways to represent Binary Relations

- Arrow Diagrams

```
1  5  
2  14
7  12

A  B
C  D
```

- Set Notation

\[ R = \{<5,A>, <2,A>, <12,A>, <12, C>, <12,D>\} \]
Ways to represent Binary Relations

- Graphs
Ways to represent Binary Relations

- Matrix Representation

\[ R = \{(2,1),(3,1),(3,2)\} \] could also be represented as:

\[
M_R = \begin{bmatrix}
1 & 2 \\
1 & 0 & 0 \\
2 & 1 & 0 \\
3 & 1 & 1
\end{bmatrix}
\]
Ternary Relations

Examples:

- Let \( R \subseteq \mathbb{Z} \times \mathbb{Z} \times \mathbb{N} \) be defined by:
  \<a, b, c> \in R \text{ if and only if } a \equiv_c b \)

  Alternate notation:
  \( R(a, b, c) \) holds if and only if \( a \equiv_c b \).

- Let \( R \subseteq \mathbb{N} \times \mathbb{N} \times \mathbb{N} \) be defined by:
  \<a, b, c> \in R \text{ iff there could be a triangle with sides of lengths } a, b \text{ and } c. \)
Unary Relations

What would a **Unary Relation** look like?

Examples?
n-ary Relations

Relations can involve any number of sets.

Example:
Let $n \in \mathbb{N}^+$
Define $R \subseteq \mathbb{R}^n$ $(\mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \ldots \times \mathbb{R})$ as:

\[ <x_1, x_2, x_3, \ldots, x_n> \in R \quad \text{if and only if} \quad \sqrt{x_1^2 + x_2^2 + x_3^2 + \ldots + x_n^2} \leq 1 \]

What is the geometric interpretation for...
n=2?
n=3?
n=1?
n=4???
## Properties of Binary Relations

<table>
<thead>
<tr>
<th>Property</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflexive</td>
<td>$(\forall a \in A) [aRa]$</td>
</tr>
<tr>
<td>Irreflexive</td>
<td>$(\forall a \in A) [\neg aRa]$</td>
</tr>
<tr>
<td>Symmetric</td>
<td>$(\forall a, b \in A) [aRb \rightarrow bRa]$</td>
</tr>
<tr>
<td>Antisymmetric</td>
<td>$(\forall a, b \in A) [aRb \land bRa \rightarrow a = b]$</td>
</tr>
<tr>
<td>Asymmetric</td>
<td>$(\forall a, b \in A) [aRb \rightarrow \neg bRa]$</td>
</tr>
<tr>
<td>Non-symmetric</td>
<td>$(\forall a, b \in A) [a \neq b \rightarrow (aRb \leftrightarrow \neg bRa)]$</td>
</tr>
<tr>
<td>Transitive</td>
<td>$(\forall a, b, c \in A) [aRb \land bRc \rightarrow aRc]$</td>
</tr>
</tbody>
</table>
Which Properties Hold?

Which of the properties on the previous slide hold for...

- $<$ over $\mathbb{R}$
- $=$ over the set $\{A, B, C\}$
- $R$ over $\mathbb{N}$ such that $aRb$ iff $a$ is a factor of $b$
- $R$ over $\mathbb{N}$ such that $aRb$ iff $a \equiv_7 b$
- $R$ over $\{\text{students in this class}\}$ such that $aRb$ iff $a$ considers $b$ to be a friend