CMSC 351: Practice Questions for Final Exam

These are practice problems for the upcoming final exam. You will be given a sheet of notes for the exam. Also, go over your homework assignments. **Warning:** This does not necessarily reflect the length, difficulty, or coverage of the actual exam.

**Problem 1.**
(a) Is $2^{n+1} = O(2^n)$?
(b) Is $2^{2n} = O(2^n)$?

**Problem 2.** Assume you have a list of $n$ elements where every number is within $k$ positions of its correct location, for some constant $k$.
(a) Give an algorithm that sorts this list with as few comparisons as possible (as a function of $n$ and $k$). Just get the high order term right. How many comparisons does your algorithm use?
(b) Show that your algorithm is optimal using a decision tree argument.

**Problem 3.** Let $A[1,..,n]$ be an array of $n$ numbers (some positive and some negative).
(a) Give an algorithm to find which three numbers have sum closest to zero. Make your algorithm as efficient as possible. Write it in pseudo-code.
(b) Analyze its running time.

**Problem 4.** A graph is tripartite if the vertices can be partitioned into three sets so that there are no edges internal to any set. The complete tripartite graph, $K(a, b, c)$, has three sets of vertices with sizes $a$, $b$, and $c$ and all possible edges between each pair of sets of vertices. $K(3,2,3)$ is pictured below. A *Hamiltonian* cycle in a graph is a cycle that traverses every vertex exactly once.

(a) For which values of $n$ does $K(1,1,n)$ have a Hamiltonian cycle. Justify your answer.
(b) For which values of $n$ does $K(1, n, n)$ have a Hamiltonian cycle. Justify your answer.
(c) For which values of $n$ does $K(n, n, n)$ have a Hamiltonian cycle. Justify your answer.

Problem 5. Let $G = (V, E)$ be an undirected graph. A triangle is a set of three vertices such that each pair has an edge.
(a) Give an efficient algorithm to find all of the triangles in a graph.
(b) How fast is your algorithm?

Problem 6. This problem is more open-ended than you would see on an exam: If you do not know how to play Sudoku, look it up. Normally, Sudoku is played on a $9 \times 9$ grid.
(a) Generalize Sudoku to larger grids.
(b) State the (generalized) Sudoku game as a decision problem.
(c) Show that the decision version of (generalized) Sudoku is in NP.
(d) Show that if you can solve the decision version of (generalized) Sudoku in polynomial time, you can solve a (generalized) Sudoku puzzle in polynomial time.

Problem 7. Consider a game played on a $2 \times 2$ board with two players: Red and Blue. Each player has one piece that can be on any one square. The object is to take the opponent’s piece. A legal move is either up, down, left, or right (like a rook in chess). A piece takes the opponent’s piece by moving on top of it (like a rook in chess). The two pieces start in opposite corners. It is easy to see that this game is a win for the second player. We number the squares $(1,1)$, $(1,2)$, $(2,1)$, and $(2,2)$. Red starts in $(1,1)$ and Blue starts in $(2,2)$.

Assume we use Zobrist hashing so that for Red the squares have the following binary hash values: $(1,1)$ has 110100, $(1,2)$ has 010110, $(2,1)$ has 101001, $(2,2)$ has 001001; and for Blue $(1,1)$ has 100101, $(1,2)$ has 001101, $(2,1)$ has 110010, $(2,2)$ has 011010.

For each of the following show your calculation:
(a) What is the Zobrist hash value of the beginning position?
(b) Assume Red moves right to $(1,2)$. What is the Zobrist hash value of the new position?
(c) Assume Blue takes Red by moving up to $(1,2)$. What is the Zobrist hash value of the new position?

Problem 8. Assume you use a Bloom filter to store items in a hash table of size $s$. Each item hashes into two distinct, random locations. For each of the following show your calculation:
(a) What is the probability that you will get a false positive if there is one item in the table?
(b) What is the probability that you will get a false positive if there are two items in the table?