1. For this problem, assume that you have access to a procedure `Merge(A, B, C, m, n)`, that merges two sorted arrays `A[1..m]` and `B[1..n]` into a single sorted array `C[1..m+n]`. This procedure runs in $m + n - 1$ comparisons. Assume we also have access to unlimited dynamic array allocation.

   Suppose that you have a collection of n, 1-dimensional arrays each of length n, `D[1..n][1..n]`, such that each array is sorted (i.e. `D[i][j] <= D[i][j+1]`). We want to compute a single sorted array that contains all n^2 elements.

   The first strategy for accomplishing this is called *cascading merging*. First merge `D[1]` with `D[2]` to form an array of length $2n$, then merge this array with `D[3]` to form an array of length $3n$, etc. The final merge is between an array of length $(n-1)n$ with `D[n]` to form the final sorted array of length n^2.

   The second strategy is called *balanced merging*. Assume that n is a power of 2. First merge pairs `D[1]` and `D[2]` together, `D[3]` and `D[4]` together, and so on until merging `D[n-1]` and `D[n]` together. The result is $n/2$ sorted arrays of size $2n$ each. Next, repeat the balanced merging on these arrays, resulting in $n/4$ arrays of size $4n$ each. This is repeated until there is one array of size n^2.

   (a) What is the exact number of comparisons for cascading merging? Justify your answer. Give as simple a function $g(n)$ such that the number of comparisons is $\Theta(g(n))$.

   (b) What is the exact number of comparisons for balanced merging? Justify your answer. Give as simple a function $g(n)$ such that the number of comparisons is $\Theta(g(n))$.

   (c) Which algorithm is better, cascading merge or balanced merge (or is neither better)?

2. Assume merging two lists of sizes $m \leq n$ takes exactly $\lceil m \lg(4n/m) \rceil$ comparisons. NOTE: $n! \approx (n/e)^n$.

   (a) What is the number of comparisons for cascading merging? Just get the high order term exact. Justify your answer. Give as simple a function $g(n)$ as possible, such that the number of comparisons is $\Theta(g(n))$.

   (b) What is the number of comparisons for balanced merging? Just get the high order term exact. Justify your answer. Give as simple a function $g(n)$ as possible, such that the number of comparisons is $\Theta(g(n))$.