Verification Using Temporal Logic

Sources:

- E.M. Clarke, O. Grumberg and D. Peled. *Model Checking*. MIT Press, Cambridge, 2000.
- E.A. Emerson. “Temporal and Modal Logic.” Chapter 16 in Volume B of J. van Leeuwen, editor, *Handbook of Theoretical Computer Science*. MIT Press, Cambridge, 1990.
- D. Peled. *Software Reliability Methods*. Springer-Verlag, Berlin, 2001.
- C. Baier and J.-P. Katoen. *Principles of Model Checking*. MIT Press, Cambridge, 2008.

Temporal Logic ...

... a language for describing time-varying properties of systems.

... originally developed as a branch of philosophical logic.

... is a *modal* logic (logic of possibility and necessity).

... brought to attention of computer scientists by A. Pnueli in 1977.

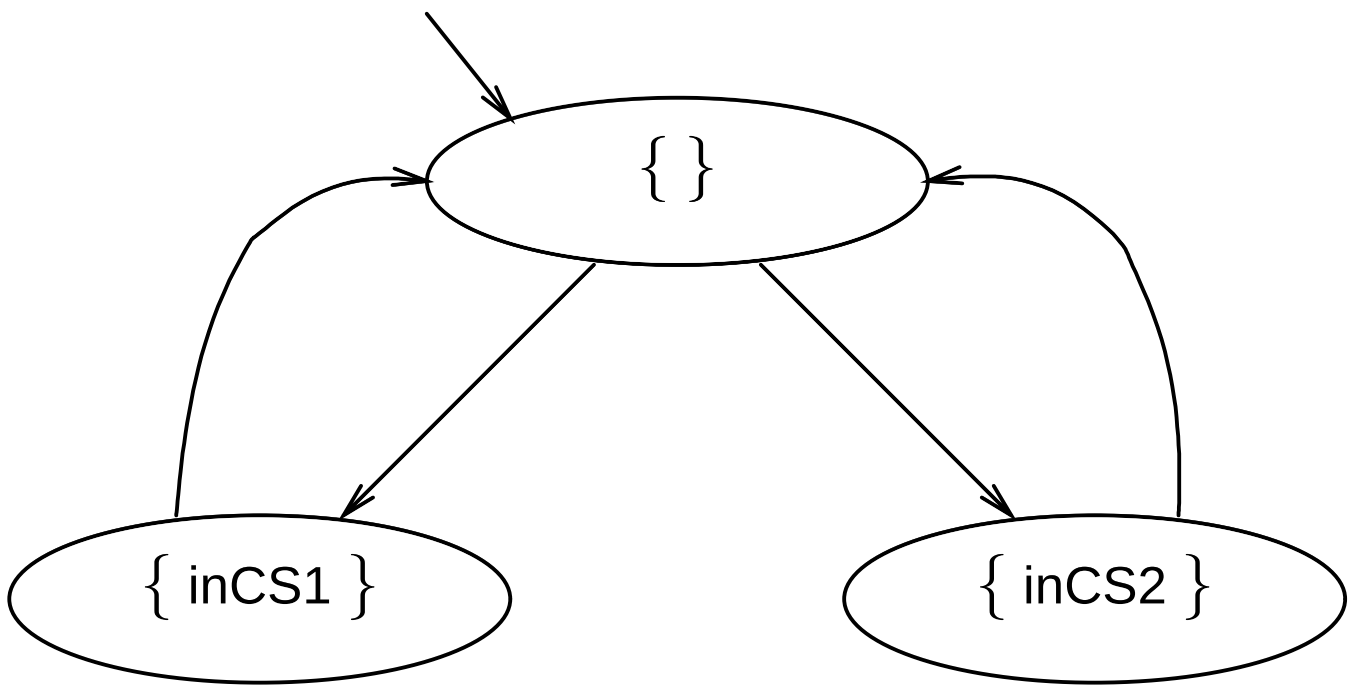
Temporal Logic Verification Framework

How is the verification question S sat R formulated in the temporal-logic setting?

- Temporal logic used to define requirements *Spec*.
- *Sys*? sat?

Kripke Structures

Kripke structures used as system models in temporal logic.

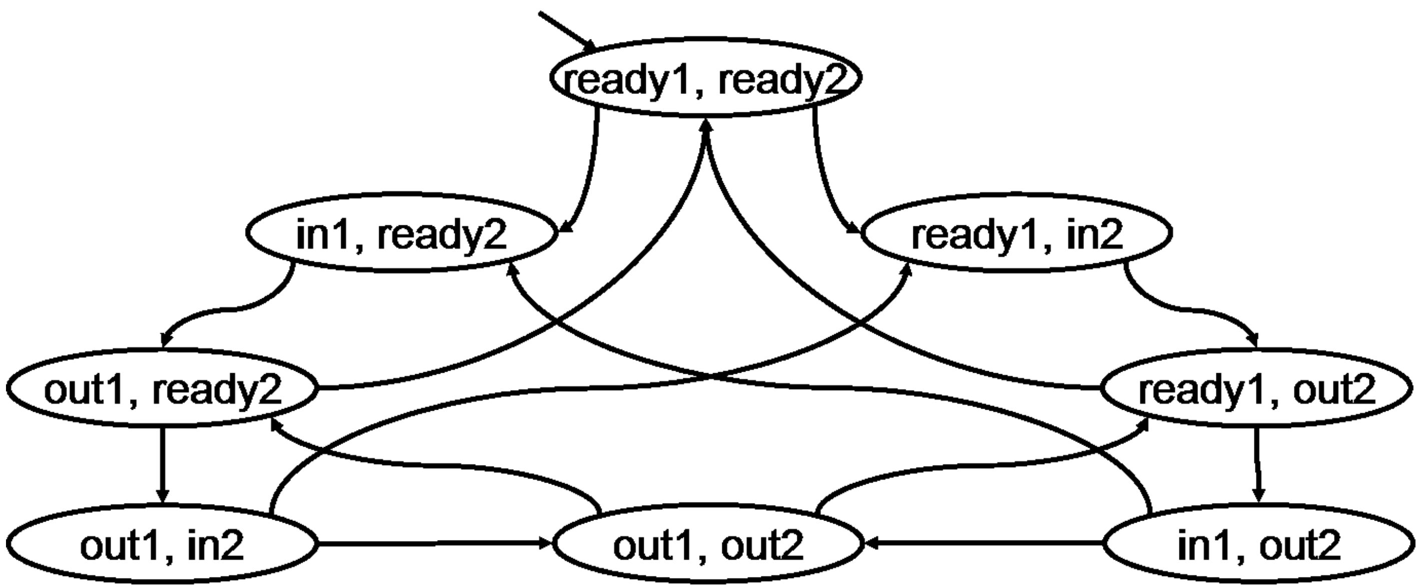


Defining Kripke Structures

Mathematically, a Kripke structure has form $\langle S, \mathcal{A}, R, \ell, s_I \rangle$, where:

- S is a set of *states*;
- $\mathcal{A}$ is a set of *atomic propositions*;
- $R \subseteq S \times S$ is the *transition relation*;
- $\ell : S \to 2^{\mathcal{A}}$ is the *labeling*; and
- $s_I \in S$ is the *initial state*.

Another Kripke Structure Example



Where Do Kripke Structures Come From?

Kripke structures are usually not specified directly by users; they are “compiled” from higher-level specifications using (small-step) *operational semantics* of programming / modeling notation.

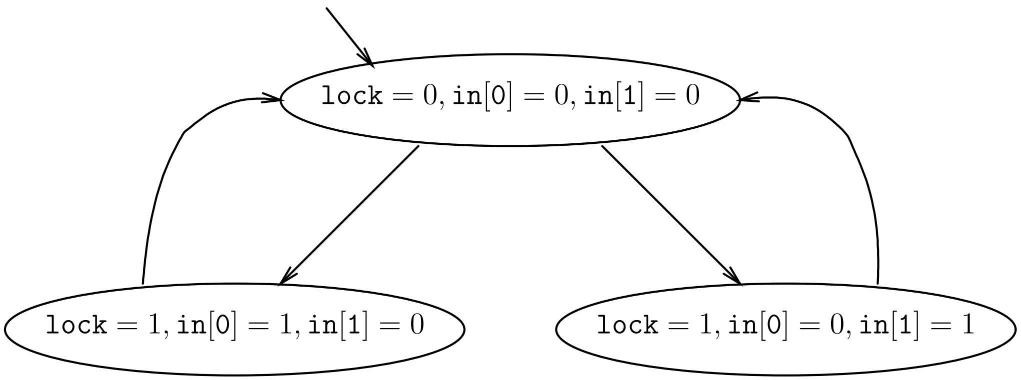
- States: assignments of variables to values
- Labeling function given as relation on states / atomic propositions
- Transitions: execution steps defined by semantics

Example: UNITY

- A modeling notation for concurrent systems
- See K.M. Chandy and J. Misra, *Parallel Program Design*, Addison Wesley, Reading MA, 1988
- Emblematic of notations used by model-checking tools

Example UNITY Model

```
program mutex
declare
    in: array [0..1] of int
    lock: int
initially
    in[0] = in[1] = lock = 0
assign
    P0enter: lock = 0 → lock, in[0] = 1, 1
    P1enter: lock = 0 → lock, in[1] = 1, 1
    P0exit: in[0] = 1 → lock, in[0] = 0,0
    P1exit: in[1] = 1 → lock, in[0] = 0,0
end
```



UNITY in Detail

- Programs consist of `declare`, `initially`, `assign` sections (among others)
- `declare` section defines variables
- `initially` section initializes variables
- `assign` section contains guarded commands
  - Guarded command has form $\mathtt{G} \rightarrow S$
  - $\mathtt{G}$ is boolean condition, S is Hoare program

Intended semantics: at each step, command with true guard is selected, executed atomically.

Program terminates when all guards are false.

Defining Temporal Logic

Now that we know what systems (*Sys*) are, what about requirements (*Spec*)?

Idea Formulas of temporal logic constitute *Spec*.

$\Rightarrow$ but there are a number of variants of temporal logic.

An important categorization: *linear time* vs. *branching time*. The difference: how time is modeled! (More on this later.)

We'll start with linear-time.

Linear-Time Temporal Logic (LTL)

Used to specify properties of *sequences*.

Given: set $(a, b \in)\mathcal{A}$ of atomic propositions. Formulas generated as follows.

$$
\begin{array}{rcll}
\phi & ::= & a & \\
& \mid & \neg\phi & \\
& \mid & \phi \vee \phi & \\
& \mid & \mathsf{X}\phi & \text{“next”} \\
& \mid & \phi \ \mathsf{U} \ \phi & \text{“until”}
\end{array}
$$

Φ_{LTL} is the set of all LTL formulas.

LTL Semantics

Formal semantics given as $\models \; \subseteq \; (2^{\mathcal{A}})^{\omega} \times \Phi_{\mathsf{LTL}}$.

- $\mathcal{A}$: set of atomic propositions
- $2^{\mathcal{A}}$: set of all subsets of $\mathcal{A}$.
- $(2^{\mathcal{A}})^{\omega}$: set of infinite sequences of $2^{\mathcal{A}}$.

That is, formulas interpreted with respect to sequences of sets of atomic propositions.

$\pi \models \phi$: "*sequence* π *has property* ϕ"

Notation

If $\pi = A_0 A_1 \ldots \in (2^{\mathcal{A}})^\omega$ then

- $\pi[i] = A_i \in 2^{\mathcal{A}}$
- $\pi[i..] = A_i A_{i+1} \ldots \in (2^{\mathcal{A}})^\omega$

LTL Semantics (cont.)

$\boxed{\pi \models a}$ if $a \in \pi[0]$.

$\boxed{\pi \models \neg\phi}$ if $\pi \not\models \phi$.

$\boxed{\pi \models \phi_1 \vee \phi_2}$ if $\pi \models \phi_1$ or $\pi \models \phi_2$.

$\boxed{\pi \models \mathsf{X}\phi}$ if $\pi[1..] \models \phi$.

$\boxed{\pi \models \phi_1 \;\; \mathsf{U} \;\; \phi_2}$ if

$\exists\, i \geq 0.\ \pi[i..] \models \phi_2$ and
$\forall\, j < i.\ \pi[j..] \models \phi_1$

Derived Operators

$$
\begin{array}{rcll}
\mathsf{tt} & \stackrel{\Delta}{=} & a \vee \neg a & \textit{true} \\
\mathsf{ff} & \stackrel{\Delta}{=} & \neg \mathsf{tt} & \textit{false} \\
\phi_1 \wedge \phi_2 & \stackrel{\Delta}{=} & \neg\big((\neg\phi_1) \vee (\neg\phi_2)\big) & \textit{conjunction} \\
\phi_1 \Rightarrow \phi_2 & \stackrel{\Delta}{=} & (\neg\phi_1) \vee \phi_2 & \textit{implication}
\end{array}
$$

More Derived Operators

Eventually $\mathsf{F}\phi$

$\mathsf{tt}\ \mathsf{U}\ \phi$

Always $\mathsf{G}\phi$

$\neg\mathsf{F}\neg\phi$

Still More Derived Operators

Release ϕ_1 R ϕ_2

$\neg((\neg\phi_1)$ U $(\neg\phi_2))$

Weak Until ϕ_1 W ϕ_2

ϕ_2 R $(\phi_1 \vee \phi_2)$

What does $\neg((\neg\phi_2)$ U $(\neg\phi_1 \wedge \neg\phi_2))$ mean?

Writing Requirements in LTL

LTL can be used to specify different properties of “execution sequences”.

$\boxed{\text{Safety}}$ “Nothing bad ever happens.”

G $\neg$*bad*

$\boxed{\text{Liveness}}$ “Something good eventually happens.”

F *good*

More Kinds of Properties

Responsiveness "Every request is eventually serviced."

$\mathsf{G}\left(\textit{req} \Rightarrow \mathsf{F}\,\textit{serviced}\right)$

Reactivity

$\left(\mathsf{GF}\,\textit{enabled}\right) \Rightarrow \left(\mathsf{GF}\,\textit{executed}\right)$

Partition

$\phi_1 \;\mathsf{U}\; \left(\phi_2 \;\mathsf{U}\; \phi_3\right)$

Example: Specifying Mutual Exclusion

$\mathcal{A} = \{\textit{in}_i, \textit{out}_i, \textit{trying}_i\}, i = 1, 2.$

Mutual Exclusion

$\mathsf{G}\,\neg(\textit{in}_1 \wedge \textit{in}_2)$

Starvation-Freedom for Process 1

$\mathsf{G}\,(\textit{trying}_1 \Rightarrow \mathsf{F}\,\textit{in}_1)$

1-Bounded Overtaking for Process 1

$\mathsf{G}\,(\textit{trying}_1 \Rightarrow (\textit{trying}_2\ \mathsf{U}\ \textit{in}_2\ \mathsf{U}\ \textit{in}_1))$

But What About System Properties?

So far we have used LTL to define properties of “executions”.

How do we use it to define properties/requirements of systems?

Idea Define a system to satisfy a formula if “all its executions” do.

How to make this precise?

Formalizing sat

Let $M = \langle S, \mathcal{A}, R, \ell, s_I \rangle$ be a Kripke structure.

- $\pi = s_0 s_1 \ldots \in S^\omega$ is an *execution* of M if:
  - $s_0 = s_I$
  - For all $i \geq 0$, $\langle s_i, s_{i+1} \rangle \in R$ or for all $s \in S$, $\langle s_i, s \rangle \notin R$ and $s_{i+1} = s_i$.

  $E(M)$: set of all executions of M.
- $\ell(s_0 s_1 \ldots) = \ell(s_0)\ell(s_1)\ldots$.
- M sat ϕ if for all $\pi \in E(M), \ell(\pi) \models \phi$.

What We Have...

... a verification framework based on LTL!

Systems:	Kripke structures
Requirements:	LTL formulas
sat:	sat

What's next?

1. Other temporal logics.
2. How to show M sat ϕ.

Branching-Time Temporal Logic

Recall:

- Two kinds of temporal logic: linear-time and branching-time
- Linear-time: models are sequences
- Branching-time: models are “trees” (alternatively, states in Kripke structures)

How do we define a branching-time temporal logic?

CTL^* = LTL $+$ Path Quantifiers

The CTL^* approach: add “path quantifiers” to LTL!

Eϕ: satisfied by a state if there exists a path from the state and satisfying ϕ.

E is a *path quantifier*.

Syntax of CTL*

Defines *state* formulas ...

$$\begin{array}{rcl} \sigma & ::= & a \\ & \mid & \neg\sigma \\ & \mid & \sigma \vee \sigma \\ & \mid & \mathsf{E}\phi \end{array}$$

Σ_{CTL^*}: set of all CTL* (state) formulas.

Syntax of CTL* (cont.)

... and *path* formulas.

$$
\begin{array}{rcl}
\phi & ::= & \sigma \\
 & | & \neg\phi \\
 & | & \phi \vee \phi \\
 & | & \mathsf{X}\,\phi \\
 & | & \phi\ \mathsf{U}\ \phi
\end{array}
$$

Φ_{CTL^*}: set of all CTL* path formulas.

Defining Semantics of CTL*

... given with respect to Kripke structure $M = \langle S, \mathcal{A}, R, \ell, s_I \rangle$ as relation

$$\models_M \subseteq (S \times \Sigma_{\textsf{CTL}^*}) \cup (S^\omega \times \Phi_{\textsf{CTL}^*})$$

... i.e. states related to state formulas and paths to path formulas.

Fix M in what follows.

Notation $E(M, s) \subseteq S^\omega$: execution paths emanating from s. $\pi \in E(M, s)$ if:

- $\pi[0] = s$
- For all $i \geq 0$, $\langle \pi[i], \pi[i+1] \rangle \in R$ or for all $s' \in S$, $\langle \pi[i], s' \rangle \notin R$ and $\pi[i+1] = \pi[i]$.

Semantics of State Formulas

$\boxed{s \models_M a}$ if $a \in \ell(s)$.

$\boxed{s \models_M \neg\sigma}$ if $s \not\models_M \sigma$.

$\boxed{s \models_M \sigma_1 \vee \sigma_2}$ if $s \models_M \sigma_1$ or $s \models_M \sigma_2$.

$\boxed{s \models_M \mathsf{E}\phi}$ if there exists $\pi \in E(M, s)$ such that $\pi \models_M \phi$.

Semantics of Path Formulas

$\boxed{\pi \models_M \sigma}$ if $\pi[0] \models_M \sigma$.

$\boxed{\pi \models_M \neg\phi}$ if $\pi \not\models_M \phi$.

$\boxed{\pi \models_M \phi_1 \vee \phi_2}$ if $\pi \models_M \phi_1$ or $\pi \models_M \phi_2$.

$\boxed{\pi \models_M \mathsf{X}\phi}$ if $\pi[1..] \models_M \phi$.

$\boxed{\pi \models_M \phi_1 \ \mathsf{U} \ \phi_2}$ if $\exists\, i \geq 0.\ \sigma[i..] \models_M \phi_2$ and $\forall\, j < i.\ \sigma[j..] \models_M \phi_1$

Note: $\neg$, $\vee$, X, U as in LTL!

Derived operators: usual LTL derived operators, A $\equiv$ $\neg$E$\neg$.

CTL* Verification Framework

***Sys*: ** Kripke structures $M = \langle S, \mathcal{A}, R, \ell, s_I \rangle$

***Spec*: ** CTL* *state* formulas $\sigma \in \Sigma_{\mathsf{CTL}^*}$

sat: M sat σ if $s_I \models_M \sigma$.

Expressing Properties in CTL*

$\boxed{\text{Possibility}}$ “Any message can be lost.”

$\mathsf{AG}\left(\textit{sent} \Rightarrow \mathsf{EF}\,\textit{lost}\right)$

$\boxed{\text{LTL}}$ Any LTL system specification ϕ can be rendered as a CTL* state formula Aϕ.

So CTL* is at least as expressive a system specification notation as LTL. In fact, it is more so (LTL can’t express E).

CTL ...

... a sublogic of CTL*

... development preceded that of CTL*

... stands for “Computation Tree Logic”

... first temporal logic used in model checking

CTL formulas: state formulas in CTL* in which every path modality (U, X, G, etc.) is immediately preceded by a path quantifier.

$\boxed{\text{CTL}}$ $\mathrm{AG}(\mathit{sent} \Rightarrow \mathrm{AF}\,\mathit{received})$

$\boxed{\text{Not CTL}}$ $\mathrm{AG}(\mathit{sent} \Rightarrow \mathrm{F}\,\mathit{received})$

Formal CTL Syntax

Note: All CTL formulas are *state* formulas.

$$
\begin{array}{rcl}
\sigma & ::= & a \\
& | & \neg\sigma \\
& | & \sigma \vee \sigma \\
& | & \mathsf{EX}\,\sigma \\
& | & \mathsf{E}(\sigma\ \mathsf{U}\ \sigma) \\
& | & \mathsf{E}(\sigma\ \mathsf{R}\ \sigma)
\end{array}
$$

Σ_{CTL}: set of all CTL formulas. Recall that R is dual of U.

Derived operators: AX, AU, AR, EG, EF, AG, AF, etc.

Expressiveness of CTL System Specifications

- No more expressive than CTL*, since $\Sigma_{\text{CTL}} \subseteq \Sigma_{\text{CTL}^*}$.
- Incomparable to LTL:
  - CTL can express possibility properties.
  - LTL can express “fairness” properties.

    AFG a expressible in LTL, not in CTL.
- Consequently, strictly less expressive than CTL*! (Why?)