

1. As discussed in class, and in the book, the harmonic series, H_n , is the sum of the reciprocals of the first n numbers. The *odd harmonic series*, H_n^{odd} , is the sum of the reciprocals of the first n odd numbers.
 - (a) We found an upper bound of about $\lg(n+1)$ for the harmonic series. (See the website for a review.) Use a similar method to find an upper bound for the odd harmonic series. Assume n is a “nice” number.
 - (b) Use a similar method to find a lower bound for the odd harmonic series. Assume n is a “nice” number. NOTE: There is more than one way to solve this problem. HINT: Take the initial term, 1, out, find a lower bound, and then put the 1 back in.
 - (c) Use the integral method to find a lower bound for H_n^{odd} .
 - (d) Use the integral method to find an upper bound for H_n^{odd} .
 - (e) Give a formula for H_n^{odd} only involving the standard harmonic series H_x , which may occur more than once in the formula, where each x is a function of n . (For example, maybe, $H_n^{\text{odd}} = H_{n^2} - 2H_n + 3H_{2n}$.)
 - (f) We know that $H_n = \ln(n) + \gamma + O(1/n)$ (where γ is a constant). Using this fact and Part (e), give an asymptotic formula for H_n^{odd} .

2. Consider the function

$$5n^3 + 8n^2(\lg n)^4 + 9n$$

- (a) What is the high order term?
 - (b) What is the second order term?
 - (c) What is the third order term?
 - (d) Write the function in Theta notation (in the simplest way).
3. For each pair of expressions (A, B) below, indicate whether A is O , o , Ω , ω , or Θ of B . Note that zero, one or more of these relations may hold for a given pair; list all correct ones.

A	B
(a) $n^4/5$	$9n^3 + 1000n^2$
(a) 2^n	n^{50}
(b) \sqrt{n}	$(\log n)^{12}$
(c) \sqrt{n}	$n^{\cos(\pi n/8)}$
(d) 100^n	10^{2n}
(e) $2^{(n^2)}$	10^n
(f) $\log(n!)$	$n \log n$