Midterm Exam

This exam is closed-book and closed-notes. You may use one sheet of notes (front and back). Write all answers in the exam booklet. You may use any algorithms or results given in class. If you have a question, either raise your hand or come to the front of class. Total point value is 100 points. Good luck!

Problem 1. (25 points) Short answer questions.

(a) (8 points) Give two examples that might arise in a game implemented in Unity, one where you want a rigid-body to have a collider and one where you want a trigger.

(b) (5 points) You have three points \(p\), \(q\), and \(r\) in the plane. You want to compute a point that lies close to the center of this triangle (I don’t care exactly where). Explain how to compute such a point using the operations of affine geometry.

(c) (4 points) You have just built a terrain using Perlin noise. You want to preserve the small bumps, but you would like them to be more subdued (see the figure below).

Which of the parameters that define Perlin noise would be best to alter to affect this change? (select one)

(i) Number of layers combined to form the final function
(ii) Lacunarity (frequency increase factor)
(iii) Persistence (amplitude decrease factor)

(d) (8 points) You have a long, thin object (e.g., an arrow) that can be oriented arbitrarily in space. Which of the following collider shapes would NOT be a good choice to represent this object (Select all the apply). Briefly explain your answers.

(i) Axis-aligned bounding box (AABB)
(ii) General (arbitrarily oriented) bounding box
(iii) Bounding sphere
(iv) Capsule
Problem 2. (25 points) Your flight-combat game involves aircraft that can fly at arbitrary angles. The notions of front/back, left/right, above/below are all defined relative to the pilot flying the aircraft. (For example, if the aircraft is flying upside-down then the ground is “above” the aircraft.)

Consider an aircraft that is defined by three points \( p_1, p_2, \) and \( p_3 \), where \( p_1 \) is the nose of the aircraft, \( p_2 \) is the tip of the right wing, and \( p_3 \) is the tip of the left wing. (See the figure below.)

(a) (5 points) The three points \( p_1, p_2, \) and \( p_3 \) define a plane in three-dimensional space. (See the figure below.) Given an arbitrary point \( q \), explain how to determine whether \( q \) lies strictly above this plane, relative to the pilot’s point of view.

(b) (20 points) You want to define a camera that will follow the aircraft. Let \( p_4 \) be a point at the tail of the aircraft. The location \( c \) of the camera will be \( \ell \) units behind the aircraft (by extending the line from \( p_1 \) through \( p_4 \) for \( \ell \) units) and \( h \) units above the aircraft (along a line that is simultaneously orthogonal to \( p_1 p_4 \) and \( p_2 p_3 \)). (See the figure below.) Explain how to compute \( c \) given \( p_1, \ldots, p_4 \) and \( \ell \) and \( h \). (Hint: Start by computing three unit vectors: \( \vec{v} \) points forward, \( \vec{r} \) points to the right, and \( \vec{u} \) points up, all relative to the pilot.)

Problem 3. (25 points) Recall that a capsule is defined to be the set of points that lie within a given distance of a line segment, which is called the central axis. In class we discussed a relatively complex procedure for determining whether two capsule colliders intersect. This is much easier if the central axes of the two capsules are parallel to each other. The objective of this problem is to determine whether two capsule colliders \( C_1 \) and \( C_2 \) intersect, where the central axes of both are vertically-aligned.
For \( i \in \{1, 2\} \), capsule collider \( C_i \) has radius \( r_i \), the lower endpoint of the central axes is at \( p_i^- = (x_i, y_i^-, z_i) \), and the upper endpoint at \( p_i^+ = (x_i, y_i^+, z_i) \), where \( y_i^+ > y_i^- \). The central axis is the line segment \( s_i = p_i^- p_i^+ \). (See the figure (a).)

(a) (5 points) Let \( \ell_1 \) and \( \ell_2 \) denote the infinite vertical lines passing through \( s_1 \) and \( s_2 \), respectively. (See the figure (b).) Let \( q_1 \) and \( q_2 \) denote the points where these lines intersect the \((x, z)\)-coordinate plane. What are the coordinates of \( q_1 \) and \( q_2 \)? Given these coordinates, what is the (minimum) distance between \( \ell_1 \) and \( \ell_2 \)?

(b) (15 points) Using the result from (a) and the \( y \)-coordinates of the endpoints of the central axes, explain how to compute the distance \( \delta \) between the line segments \( s_1 \) and \( s_2 \)? (Hint: You may want to split this into cases.)

(c) (5 points) Given \( \delta \) from part (b) explain how to determine whether the two colliders intersect.

Problem 4. (25 points) NASA has asked you to help program a educational game for visualizing our solar-system. We’ll simplify the problem by considering it in 2-dimensional space.

You are given a model of Earth, which is represented using a coordinate frame located at the Earth’s center. You are given a model of the Sun, which is represented using a coordinate frame located at the Sun’s center. (See the figure below (a).) The Earth rotates around its center and it revolves around the Sun. We want to compute the coordinates of a point \( v \) on Earth after this rotation and revolution. Let’s assume a “bind pose” where the Earth is positioned 10 units away from the Sun along the \( x \)-axis.

![Diagram](a)

(a) (5 points) Let \( v_{[e]} \) denote \( v \)'s homogeneous coordinates relative to the Earth frame and let \( v_{[s]} \) denote \( v \)'s homogeneous coordinates relative to the sun’s frame. Define \( T_{s \leftarrow e} \) to be the affine transformation that maps a point in Earth-frame coordinates to the equivalent Sun-frame coordinates. Express \( T_{s \leftarrow e} \) as a \( 3 \times 3 \) matrix (assuming, as we usually do, that \( v \) is represented as a column vector).

(b) (5 points) Define \( T_{e \leftarrow s} \) to be the inverse of \( T_{s \leftarrow e} \). Express \( T_{e \leftarrow s} \) as \( 3 \times 3 \) matrix.

(c) (10 points) Let \( \text{Rot}(\theta) \) be the affine transformation that rotates space by an angle of \( \theta \), expressed as a \( 3 \times 3 \) matrix. Recall that

\[
\text{Rot}(\theta) = \begin{pmatrix}
\cos \theta & -\sin \theta & 0 \\
\sin \theta & \cos \theta & 0 \\
0 & 0 & 1
\end{pmatrix}.
\]
Explain how to obtain a transformation that maps the point \( v[e] \) (in Earth coordinates) to its updated position \( v'[s] \) (in Sun coordinates) assuming a rotation of the earth by an angle \( \phi \) and a revolution about the Sun by an angle of \( \theta \). (See the figure (b).) Express your answer as the product of a sequence of \( 3 \times 3 \) matrices.

(d) (5 points) How would your answer to (c) change if instead the input was \( v[s] \) rather than \( v[e] \)? (That is, \( v \) was given to you in Sun coordinates, not Earth coordinates.)