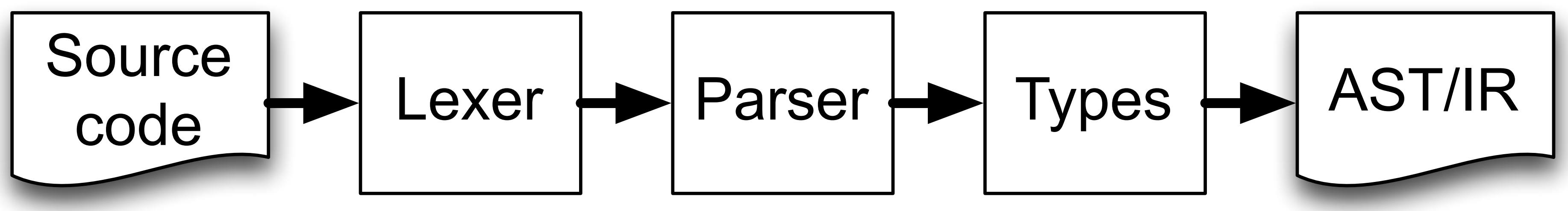
CMSC 430
Introduction to Compilers
Spring 2016

Lexing and Parsing

Overview

- Compilers are roughly divided into two parts
  - Front-end — deals with surface syntax of the language
  - Back-end — analysis and code generation of the output of the front-end



- Lexing and Parsing translate source code into form more amenable for analysis and code generation
- Front-end also may include certain kinds of semantic analysis, such as symbol table construction, type checking, type inference, etc.

Lexing vs. Parsing

- Language grammars usually split into two levels
  - Tokens — the “words” that make up “parts of speech”
    - Ex: Identifier [a-zA-Z_]+
    - Ex: Number [0-9]+
  - Programs, types, statements, expressions, declarations, definitions, etc — the “phrases” of the language
    - Ex: if (expr) expr;
    - Ex: def id(id, ..., id) expr end
- Tokens are identified by the lexer
  - Regular expressions
- Everything else is done by the parser
  - Uses grammar in which tokens are primitives
  - Implementations can look inside tokens where needed

Lexing vs. Parsing (cont'd)

- Lexing and parsing often produce abstract syntax tree as a result
  - For efficiency, some compilers go further, and directly generate intermediate representations

- Why separate lexing and parsing from the rest of the compiler?
- Why separate lexing and parsing from each other?

Parsing theory

- Goal of parsing: Discovering a parse tree (or derivation) from a sentence, or deciding there is no such parse tree
- There's an alphabet soup of parsers
  - Cocke-Younger-Kasami (CYK) algorithm; Earley's Parser
    - Can parse *any* context-free grammar (but inefficient)
  - LL(k)
    - top-down, parses input left-to right (first L), produces a leftmost derivation (second L), k characters of lookahead
  - LR(k)
    - bottom-up, parses input left-to-right (L), produces a rightmost derivation (R), k characters of lookahead
- We will study only some of this theory
  - But we'll start more concretely

Parsing practice

- Yacc and lex — most common ways to write parsers
  - yacc = “yet another compiler compiler” (but it makes parsers)
  - lex = lexical analyzer (makes lexers/tokenizers)
- These are available for most languages
  - bison/flex — GNU versions for C/C++
  - ocamlyacc/ocamllex — what we’ll use in this class

Example: Arithmetic expressions

- High-level grammar:
  - $E \rightarrow E + E \mid n \mid (E)$
- What should the tokens be?
  - Typically they are the terminals in the grammar
    - {+, (,), n}
    - Notice that n itself represents a set of values
    - Lexers use *regular expressions* to define tokens
  - But what will a typical input actually look like?

1	+	2	+	\n	(	3		+		4	2	)	eof

- We probably want to allow for whitespace
  - Notice not included in high-level grammar: lexer can discard it
- Also need to know when we reach the end of the file
  - The parser needs to know when to stop

Lexing with ocamllex (.mll)

```
(* Slightly simplified format *)
{ header }
rule entrypoint = parse
        regexp_1 { action_1 }
      | …
      | regexp_n { action_n }
and …
{ trailer }
```

- Compiled to .ml output file
  - header and trailer are inlined into output file as-is
  - regexps are combined to form one (big!) finite automaton that recognizes the union of the regular expressions
    - Finds *longest* possible match in the case of multiple matches
    - Generated regexp matching function is called entrypoint

Lexing with ocamllex (.mll)

```
(* Slightly simplified format *)
{ header }
rule entrypoint = parse
        regexp_1 { action_1 }
      | …
      | regexp_n { action_n }
and …
{ trailer }
```

- When match occurs, generated entrypoint function returns value in corresponding action
  - If we are lexing for ocamlyacc, then we'll return tokens that are defined in the ocamlyacc input grammar

Example

```
{
  open Ex1_parser
  exception Eof
}
rule token = parse
    [' ' '\t' '\r']       { token lexbuf }  (* skip blanks *)
  | ['\n' ]               { EOL }
  | ['0'-'9']+ as lxm     { INT(int_of_string lxm) }
  | '+'                   { PLUS }
  | '('                   { LPAREN }
  | ')'                   { RPAREN }
  | eof                   { raise Eof }
```

```
(* token definition from Ex1_parser *)
type token =
  | INT of (int)
  | EOL
  | PLUS
  | LPAREN
  | RPAREN
```

Generated code

```
# 1 "ex1_lexer.mll"  (* line directives for error msgs *)

  open Ex1_parser
  exception Eof

# 7 "ex1_lexer.ml"
let __ocaml_lex_tables = {...}  (* table-driven automaton *)
let rec token lexbuf = ...  (* the generated matching fn *)
```

- You don't need to understand the generated code
  - But you should understand it's not magic
- Uses Lexing module from OCaml standard lib
- Notice that token rule was compiled to token fn
  - Mysterious lexbuf from before is the argument to token
  - Type can be examined in Lexing module ocamldoc

Lexer limitations

- Automata limited to 32767 states
  - Can be a problem for languages with lots of keywords

```
rule token = parse
  "keyword_1"   { ... }
| "keyword_2"   { ... }
| ...
| "keyword_n" { ... }
| ['A'-'Z' 'a'-'z'] ['A'-'Z' 'a'-'z' '0'-'9' '_'] * as id
                 { IDENT id}
```

  - Solution?

Parsing

- Now we can build a parser that works with lexemes (tokens) from token.mll
  - Recall from 330 that parsers work by consuming one character at a time off input while building up parse tree
  - Now the input stream will be tokens, rather than chars

1	+	2	+	\n	(	3		+		4	2	)	eof

INT(1)	PLUS	INT(2)	PLUS	LPAREN	INT(3)	PLUS	INT(42)	RPAREN	eof

  - Notice parser doesn't need to worry about whitespace, deciding what's an INT, etc

Suitability of Grammar

- Problem: our grammar is ambiguous
  - $E \rightarrow E + E \mid n \mid (E)$
  - Exercise: find an input that shows ambiguity
- There are parsing technologies that can work with ambiguous grammars
  - But they'll provide multiple parses for ambiguous strings, which is probably not what we want
- Solution: remove ambiguity
  - One way to do this from 330:
  - $E \rightarrow T \mid E + T$
  - $T \rightarrow n \mid (E)$

Parsing with ocamlyacc (.mly)

```
%{
  header
%}
  declarations
%%
  rules
%%
  trailer
```

.mly input

```
type token =
   | INT of (int)
   | EOL
   | PLUS
   | LPAREN
   | RPAREN

val main :
   (Lexing.lexbuf  -> token) ->
                Lexing.lexbuf -> int
```

.mli output

- Compiled to .ml and .mli files
  - .mli file defines token type and entry point main for parsing
    - Notice first arg to main is a fn from a lexbuf to a token, i.e., the function generated from a .mll file!

Parsing with ocamlyacc (.mly)

```
%{
  header
%}
  declarations
%%
  rules
%%
  trailer
```

`.mly` input

```
(* header *)
type token = ...
...
let yytables = ...
(* trailer *)
```

`.ml` output

- .ml file uses Parsing library to do most of the work
  - header and trailer copied direct to output
  - declarations lists tokens and some other stuff
  - rules are the productions of the grammar
    - Compiled to yytables; this is a table-driven parser Also include *actions* that are executed as parser executes
    - We'll see an example next

Actions

- In practice, we don't just want to check whether an input parses; we also want to do something with the result
  - E.g., we might build an AST to be used later in the compiler
- Thus, each production in ocamlyacc is associated with an *action* that produces a result we want
- Each rule has the format
  - lhs: rhs {act}
  - When parser uses a production lhs → rhs in finding the parse tree, it runs the code in act
  - The code in act can refer to results computed by actions of other non-terminals in rhs, or token values from terminals in rhs

Example

```
%token <int> INT
%token EOL PLUS LPAREN RPAREN
%start main                 /* the entry point */
%type <int> main
%%
main:
| expr EOL                  { $1 }          (* 1 *)
expr:
| term                      { $1 }          (* 2 *)
| expr PLUS term            { $1 + $3 }     (* 3 *)
term:
| INT                       { $1 }          (* 4 *)
| LPAREN expr RPAREN        { $2 }          (* 5 *)
```

- Several kinds of declarations:
  - %token — define a token or tokens used by lexer
  - %start — define start symbol of the grammar
  - %type — specify type of value returned by actions

Actions, in action

INT(1)	PLUS	INT(2)	PLUS	LPAREN	INT(3)	PLUS	INT(42)	RPAREN	eof

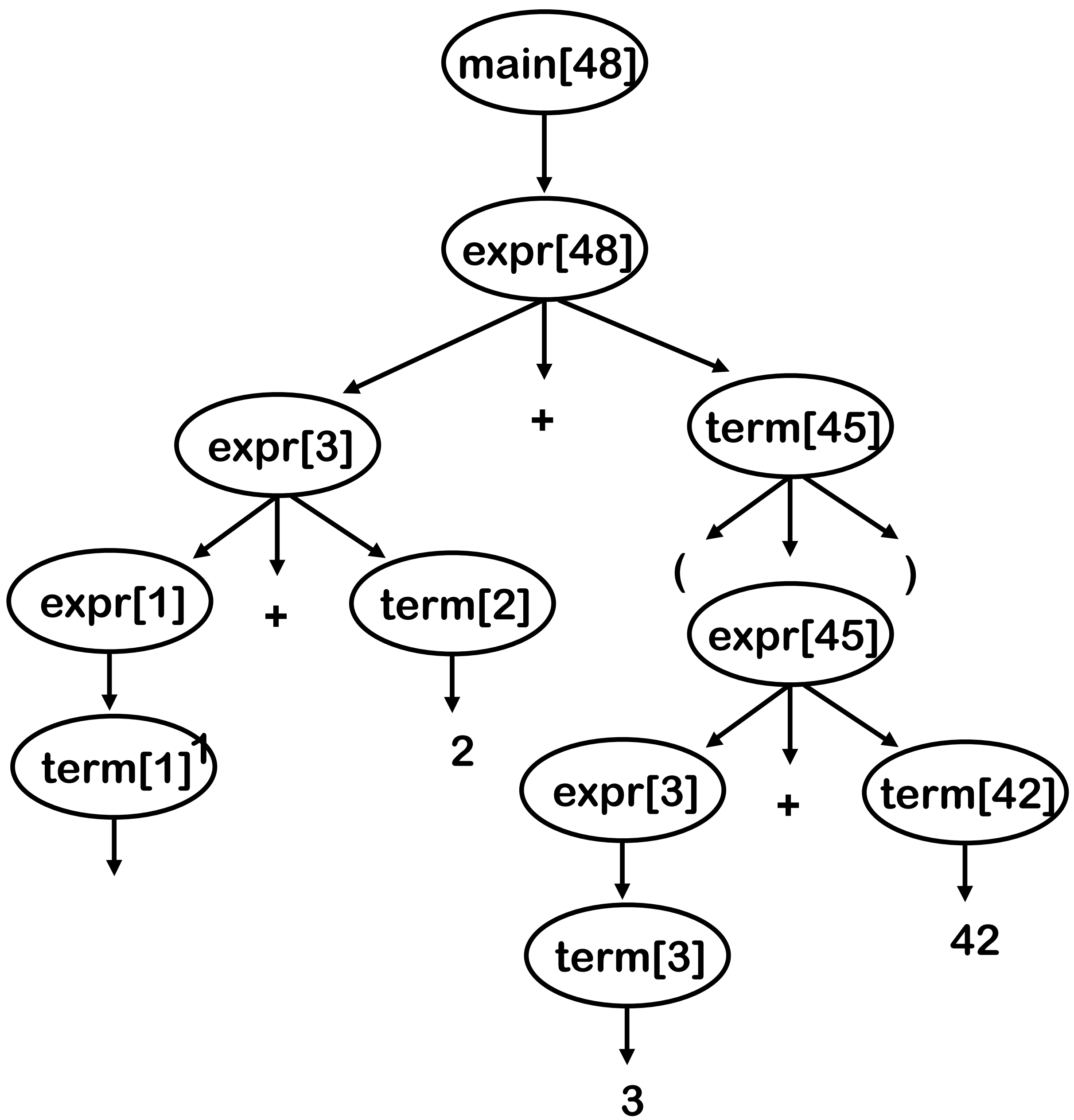
. 1+2+(3+42)$
term[1].+2+(3+42)$
expr[1].+2+(3+42)$
expr[1]+term[2].+(3+42)$
expr[3].+(3+42)$
expr[3]+(term[3].+42)$
expr[3]+(expr[3].+42)$
expr[3]+(expr[3]+term[42].)$
expr[3]+(expr[45].)$
expr[3]+term[45].$
expr[48].$
main[48]

```
main:
| expr EOL                { $1 }
expr:
| term                    { $1 }
| expr PLUS term          { $1 + $3 }
term:
| INT                     { $1 }
| LPAREN expr RPAREN      { $2 }
```

- The "." indicates where we are in the parse
    - We've skipped several intermediate steps here, to focus only on actions
    - (Details next)

Actions, in action

INT(1)	PLUS	INT(2)	PLUS	LPAREN	INT(3)	PLUS	INT(42)	RPAREN	eof




```
main:
| expr EOL              { $1 }
expr:
| term                  { $1 }
| expr PLUS term        { $1 + $3 }
term:
| INT                   { $1 }
| LPAREN expr RPAREN    { $2 }
```

Invoking lexer/parser

```
try
  let lexbuf = Lexing.from_channel stdin in
    while true do
      let result = Ex1_parser.main Ex1_lexer.token lexbuf in
        print_int result; print_newline(); flush stdout
    done
with Ex1_lexer.Eof ->
  exit 0
```

- Tip: can also use Lexing.from_string and Lexing.from_function

Terminology review

- Derivation
  - A sequence of steps using the productions to go from the start symbol to a string
- Rightmost (leftmost) derivation
  - A derivation in which the rightmost (leftmost) nonterminal is rewritten at each step
- Sentential form
  - A sequence of terminals and non-terminals derived from the start-symbol of the grammar with 0 or more reductions
  - I.e., some intermediate step on the way from the start symbol to a string in the language of the grammar
- Right- (left-)sentential form
  - A sentential form from a rightmost (leftmost) derivation
- FIRST(α)
  - Set of initial symbols of strings derived from α

Bottom-up parsing

- ocamlyacc builds a bottom-up parser
  - Builds derivation from input back to start symbol

$$S \Rightarrow \gamma 0 \Rightarrow \gamma 1 \Rightarrow \gamma 2 \Rightarrow \ldots \Rightarrow \gamma n{-}1 \Rightarrow \gamma n \Rightarrow \text{input}$$

$\xleftarrow{\hspace{6em}}$ bottom-up

- To reduce γi to $\gamma i{-}1$
  - Find production $A \rightarrow \beta$ where β is in γi, and replace β with A
- In terms of parse tree, working from leaves to root
  - Nodes with no parent in a partial tree form its *upper fringe*
  - Since each replacement of β with A shrinks upper fringe, we call it a reduction.
- Note: need not actually build parse tree
  - |parse tree nodes| = |input| + |reductions|

Bottom-up parsing, illustrated

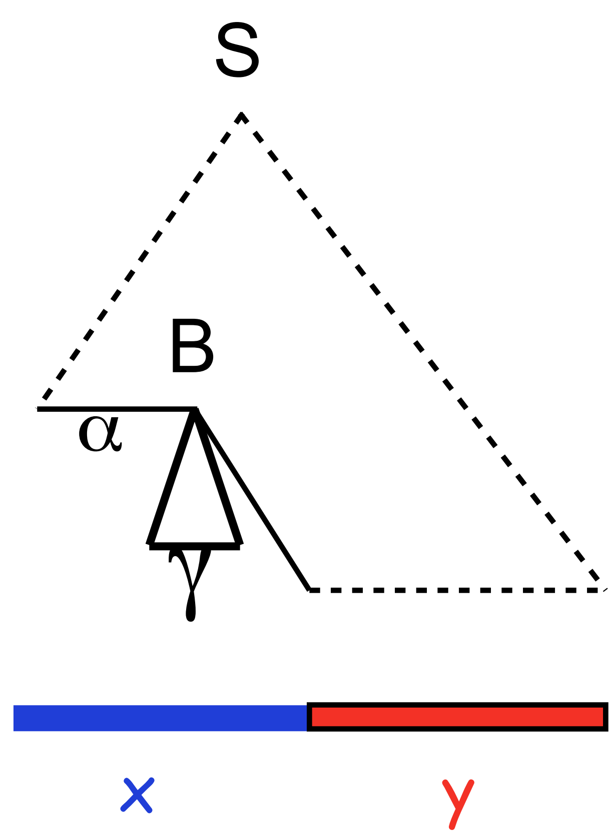
LR(1) parsing

- Scan input left-to-right
- Rightmost derivtaion
- 1 token lookahead

rule $B \rightarrow \gamma$

$$S \Rightarrow^* \alpha\ B\ y \Rightarrow \alpha\ \gamma\ y \Rightarrow^* x\ y$$

Upper fringe: solid
Yet to be parsed: dashed



Bottom-up parsing, illustrated

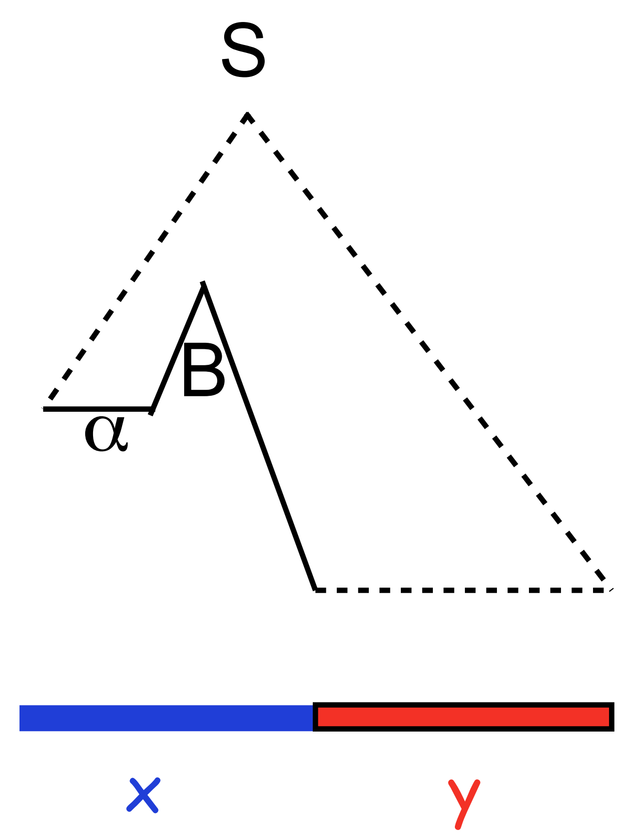
LR(1) parsing

- Scan input left-to-right
- Rightmost derivtaion
- 1 token lookahead

rule $B \rightarrow \gamma$

$$S \Rightarrow^* \alpha\, B\, y \Rightarrow \alpha\, \gamma\, y \Rightarrow^* x\, y$$

Upper fringe: solid
Yet to be parsed: dashed



Finding reductions

- Consider the following grammar

  1. S → a A B e
  2. A → A b c
  3. | b
  4. B → d

  Input: abbcde

Sentential Form	Production	Position
abbcde	3	2
aAbcde	2	4
aAde	4	3
aABe	1	4
S	N/A	N/A

- How do we find the next reduction?
  - How do we do this efficiently?

Handles

- Goal: Find substring β of tree's frontier that matches some production $A \rightarrow \beta$
  - (And that occurs in the rightmost derivation)
  - Informally, we call this substring β a *handle*
- Formally,
  - A *handle* of a right-sentential form γ is a pair $(A\rightarrow\beta,k)$ where
    - $A\rightarrow\beta$ is a production and k is the position in γ of β's rightmost symbol.
    - If $(A\rightarrow\beta,k)$ is a handle, then replacing β at k with A produces the right sentential form from which γ is derived in the rightmost derivation.
  - Because γ is a right-sentential form, the substring to the right of a handle contains only terminal symbols
    - $\Rightarrow$ the parser doesn't need to scan past the handle (only lookahead)

Example

- Grammar

1. S → E
2. E → E + T
3. | E - T
4. | T
5. T → T * F
6. | T / F
7. | F
8. F → n
9. | id
10. | (E)

Production	Sentential Form	Handle (prod,k)
	S	
1	E	1,1
3	E-T	3,3
5	E-T*F	5,5
9	E-T*id	9,5
7	E-F*id	7,3
8	E-n*id	8,3
4	T-n*id	4,1
7	F-n*id	7,1
9	id-n*id	9,1

Handles for rightmost derivation of id-n*id

Finding reductions

- Theorem: If G is unambiguous, then every right-sentential form has a unique handle
  - If we can find those handles, we can build a derivation!
- Sketch of Proof:
  - G is unambiguous $\Rightarrow$ rightmost derivation is unique
  - $\Rightarrow$ a unique production $A \rightarrow \beta$ applied to derive γi from $\gamma i–1$
  - and a unique position k at which $A \rightarrow \beta$ is applied
  - $\Rightarrow$ a unique handle $(A \rightarrow \beta, k)$
- This all follows from the definitions

Bottom-up handle pruning

- *Handle pruning*: discovering handle and reducing it
  - Handle pruning forms the basis for bottom-up parsing
- So, to construct a rightmost derivation

$$S \Rightarrow \gamma_0 \Rightarrow \gamma_1 \Rightarrow \gamma_2 \Rightarrow \ldots \Rightarrow \gamma_{n-1} \Rightarrow \gamma_n \Rightarrow \text{input}$$

- Apply the following simple algorithm

for $i \leftarrow n$ to 1 by -1

  Find handle $(A_i \rightarrow \beta_i , k_i)$ in γ_i

  Replace β_i with A_i to generate γ_{i-1}

  - This takes $2n$ steps

Shift-reduce parsing algorithm

- Maintain a stack of terminals and non-terminals matched so far
  - Rightmost terminal/non-terminal on top of stack
  - Since we're building rightmost derivation, will look at top elements of stack for reductions

```
push INVALID
token ← next_token( )
repeat until (top of stack = Goal and token = EOF)
    if the top of the stack is a handle A→β
        then      // reduce β to A
            pop |β| symbols off the stack
            push A onto the stack
        else if (token ≠ EOF)
            then // shift
                push token
                token ← next_token( )
            else      // need to shift, but out of input
                report an error
```

Potential errors

- Can't find handle
- Reach end of file

Example

1. Shift until the top of the stack is the right end of a handle
2. Find the left end of the handle & reduce

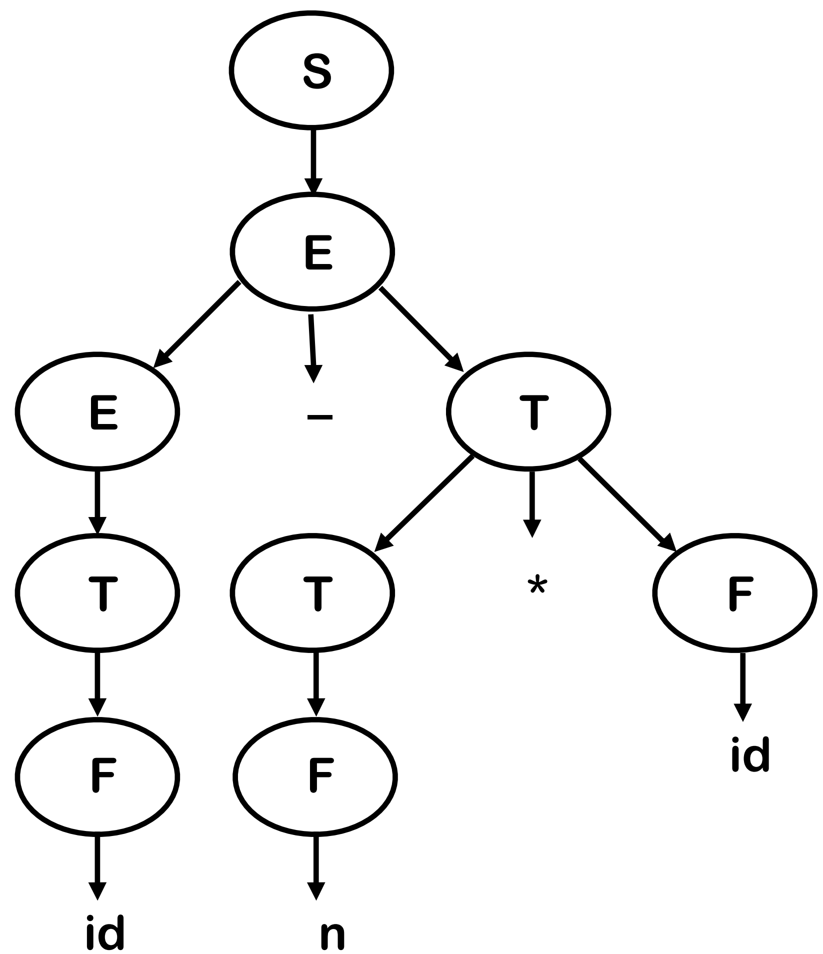
- Grammar

1. S → E
2. E → E + T
3. | E - T
4. | T
5. T → T * F
6. | T / F
7. | F
8. F → n
9. | id
10. | (E)

Shift/reduce parse of id-n*id

Stack	Input	Handle (prod,k)	Action
	id-n*id	none	shift
id	-n*id	9,1	reduce 9
F	-n*id	7,1	reduce 7
T	-n*id	4,1	reduce 4
E	-n*id	none	shift
E-	n*id	none	shift
E-n	*id	8,3	reduce 8
E-F	*id	7,3	reduce 7
E-T	*id	none	shift
E-T*	id	none	shift
E-T*id		9,5	reduce 9
E-T*F		5,5	reduce 5
E-T		3,3	reduce 3
E		1,1	reduce 1
S		none	accept

Parse tree for example



Algorithm actions

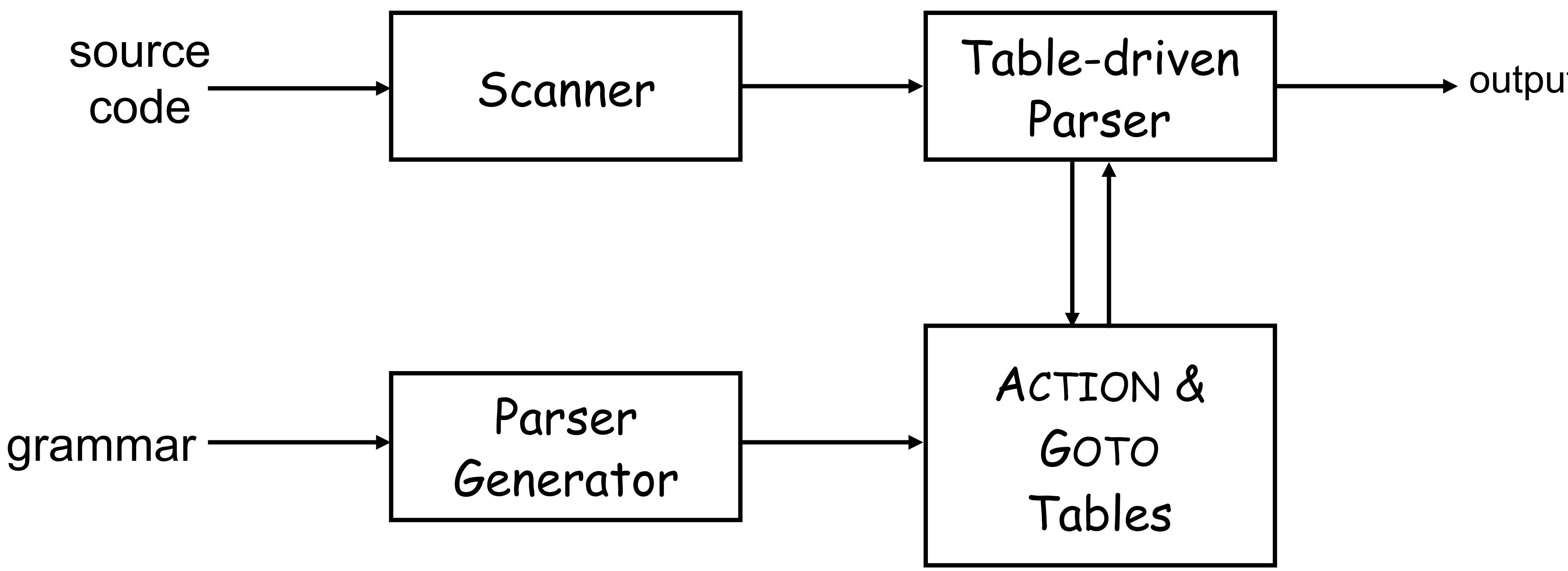
- Shift-reduce parsers have just four actions
  - Shift — next word is shifted onto the stack
  - Reduce — right end of handle is at top of stack
    - Locate left end of handle within the stack
    - Pop handle off stack and push appropriate lhs
  - Accept — stop parsing and report success
  - Error — call an error reporting/recovery routine
- Cost of operations
  - Accept is constant time
  - Shift is just a push and a call to the scanner
  - Reduce takes |rhs| pops and 1 push
    - If handle-finding requires state, put it in the stack ⇒ 2x work
  - Error depends on error recovery mechanism

Finding handles

- To be a handle, a substring of sentential form γ must :
  - Match the right hand side β of some rule $A \rightarrow \beta$
  - There must be some rightmost derivation from the start symbol that produces γ with $A \rightarrow \beta$ as the last production applied
  - $\Rightarrow$ Looking for rhs's that match strings is not good enough
- How can we know when we have found a handle?
  - LR(1) parsers use DFA that runs over stack and finds them
    - One token look-ahead determines next action (shift or reduce) in each state of the DFA.
  - A grammar is LR(1) if we can build an LR(1) parser for it
- LR(0) parsers: no look-ahead

LR(1) parsing

- Can use a set of tables to describe LR(1) parser



- ocamlyacc automates the process of building the tables
  - Standard library Parser module interprets the tables
- LR parsing invented in 1965 by Donald Knuth
- LALR parsing invented in 1969 by Frank DeRemer

LR(1) parsing algorithm

```
stack.push(INVALID); stack.push(s0);
not_found = true;
token = scanner.next_token();
do while (not_found) {
  s = stack.top();
  if ( ACTION[s,token] == "reduce A→β" ) {
    stack.popnum(2*|β|); // pop 2*|β| symbols
    s = stack.top();
    stack.push(A);
    stack.push(GOTO[s,A]);
  }
  else if ( ACTION[s,token] == "shift si" ) {
    stack.push(token); stack.push(si);
    token ← scanner.next_token();
  }
  else if ( ACTION[s,token] == "accept" && token == EOF )
     not_found = false;
  else report a syntax error and recover;
}
report success;
```

- Two tables
  - ACTION: reduce/shift/accept
  - GOTO: state to be in after reduce
- Cost
  - |input| shifts
  - |derivation| reductions
  - One accept
- Detects errors by failure to shift, reduce, or accept

Example parser table

- ocamlyacc -v ex1_parser.mly — produce .output file with parser table

state	action						goto			productions
	.	EOL	+	N	(	)	main	expr	term	
0										*(special)*
1				s3	s4		acc	6	7	*entry* → . main
2										*(special)*
3	r4									term → INT .
4				s3	s4			8	7	term → (. expr)
5										*(special)*
6		s9	s10							main → expr . EOL | expr → expr . + term
7	r2									expr → term .
8			s10			s11				expr → expr . + term | term → (expr .)
9	r1									main → expr EOL .
10				s3	s4				12	expr → expr + . term
11	r5									term → (expr) .
12	r3									expr → expr + term .

NB: Numbers in shift refer to state numbers

Numbers in reduction refer to production numbers

Example parse (N+N+N)

Stack	Input	Action
1	N+N+N	s3
1,N,3	+N+N	r4
1,term,7	+N+N	r2
1,expr,6	+N+N	s10
1,expr,6,+,10	N+N	s3
1,expr,6,+,10,N,3	+N	r4
1,expr,6,+,10,term,12	+N	r3
1,expr,6	+N	s10
1,expr,6,+,10	N	s3
1,expr,6,+,10,N,3		r4
1,expr,6,+,10,term,12		r3
1,expr,6		s9
1,expr,6,EOL,9		r1
accept		

Example parser table (cont'd)

- Notes
  - Notice derivation is built up (bottom to top)
  - Table only contains kernel of each state
    - Apply closure operation to see all the productions in the state
- LR(1) parsing requires start symbol not on any rhs
  - Thus, ocamlyacc actually adds another production
    - %entry% → \001 main
    - (so the acc in the previous table is a slight fib)
- Values returned from actions stored on the stack
  - Reduce triggers computation of action result

Why does this work?

- Stack = upper fringe
  - So all possible handles on top of stack
  - Shift inputs until top elements of stack form a handle
- Build a handle-recognizing DFA
  - Language of handles is regular
  - ACTION and GOTO tables encode the DFA
    - Shift = DFA transition
    - Reduce = DFA accept
      - New state = GOTO[state at top of stack (afetr pop), lhs]
- If we can build these tables, grammar is LR(1)

LR(k) items

- An *LR(k) item* is a pair [P, δ], where
  - P is a production A→β with a • at some position in the rhs
  - δ is a lookahead string of length ≤ k (words or $)
  - The • in an item indicates the position of the top of the stack
- LR(1):
  - [A→•βγ,a] — input so far consistent with using A →βγ immediately after symbol on top of stack
  - [A →β•γ,a] — input so far consistent with using A →βγ at this point in the parse, and parser has already recognized β
  - [A →βγ•,a] — parser has seen βγ, and lookahead of a consistent with reducing to A
- LR(1) items represent valid configurations of an LR(1) parser; DFA states are sets of LR(1) items

LR(k) items, cont'd

- Ex: $A \to BCD$ with lookahead a can yield 4 items
  - $[A \to \bullet BCD, a]$, $[A \to B \bullet CD, a]$, $[A \to BC \bullet D, a]$, $[A \to BCD \bullet, a]$
  - Notice: set of LR(1) items for a grammar is finite
- Carry lookaheads along to choose correct reduction
  - Lookahead has no direct use in $[A \to \beta \bullet \gamma, a]$
  - In $[A \to \beta \bullet, a]$, a lookahead of $a \Rightarrow$ reduction by $A \to \beta$
  - For $\{ [A \to \beta \bullet, a], [B \to \gamma \bullet \delta, b] \}$
    - Lookahead of $a \Rightarrow$ reduce to A
    - $\text{FIRST}(\delta) \Rightarrow$ shift
    - (else error)

LR(1) table construction

- States of LR(1) parser contain sets of LR(1) items
  - Initial state s0
    - Assume S' is the start symbol of grammar, does not appear in rhs
      - (Extend grammar if necessary to ensure this)
    - s0 = closure([S' →•S,$])   ($ = EOF)
  - For each sk and each terminal/non-terminal X, compute new state goto(sk,X)
    - Use closure() to "fill out" kernel of new state
    - If the new state is not already in the collection, add it
    - Record all the transitions created by goto( )
      - These become ACTION and GOTO tables
      - i.e., the handle-finding DFA
  - This process eventually reaches a fixpoint

Closure()

- $[A \rightarrow \beta \bullet B\delta, a]$ implies $[B \rightarrow \bullet \gamma, x]$ for each production with B on lhs and each $x \in \text{FIRST}(\delta a)$
  - (If you're about to see a B, you may also see a γ)

```
Closure( s )
  while ( s is still changing )
    ∀ items [A → β •Bδ,a] ∈ s            // item with • to left of nonterminal B
      ∀ productions  B → γ ∈ P           // all productions for B
        ∀ b  ∈ FIRST(δa)                 // tokens appearing after B
          if  [B → • γ,b] ∉ s            // form LR(1) item w/ new lookahead
            then add [B→ • γ,b] to s     // add item to s if new
```

- Classic fixed-point method
- Halts because $s \subset$ ITEMS (worklist version is faster)
  - Closure "fills out" a state

Example — closure with LR(0)

$S \to E$

$E \to T{+}E$

$\quad\mid\ T$

$T \to id$

[kernel item]
[derived item]

$[S \to \bullet\ E]$
$[E \to \bullet\ T{+}E]$
$[E \to \bullet\ T]$
$[T \to \bullet\ id]$

$[E \to T{+}\ \bullet\ E]$
$[E \to \bullet\ T{+}E]$
$[E \to \bullet\ T]$
$[T \to \bullet\ id]$

Example — closure with LR(1)

$S \rightarrow E$

$E \rightarrow T+E$

$\quad | \ T$

$T \rightarrow id$

[kernel item]
[derived item]

[S → • E, $]
[E → • T+E, $]
[E → • T, $]
[T → • id, +]
[T → • id, $]

[E → T+ • E, $]
[E → • T+E, $]
[E → • T, $]
[T → • id, +]
[T → • id, $]

Goto

- Goto(s,x) computes the state that the parser would reach if it recognized an x while in state s
  - Goto({ [$A \rightarrow \beta \bullet X\delta$,a] }, X) produces [$A \rightarrow \beta X \bullet \delta$,a]
  - Should also includes closure([$A \rightarrow \beta X \bullet \delta$,a])

```
Goto( s, X )
   new ←Ø
   ∀ items [A→β•Xδ,a] ∈ s          // for each item with • to left of X
      new ← new ∪ [A→βX•δ,a]       // add item with • to right of X

   return closure(new)              // remember to compute closure!
```

- Not a fixed-point method!
- Straightforward computation
- Uses closure ( )
  - Goto() moves forward

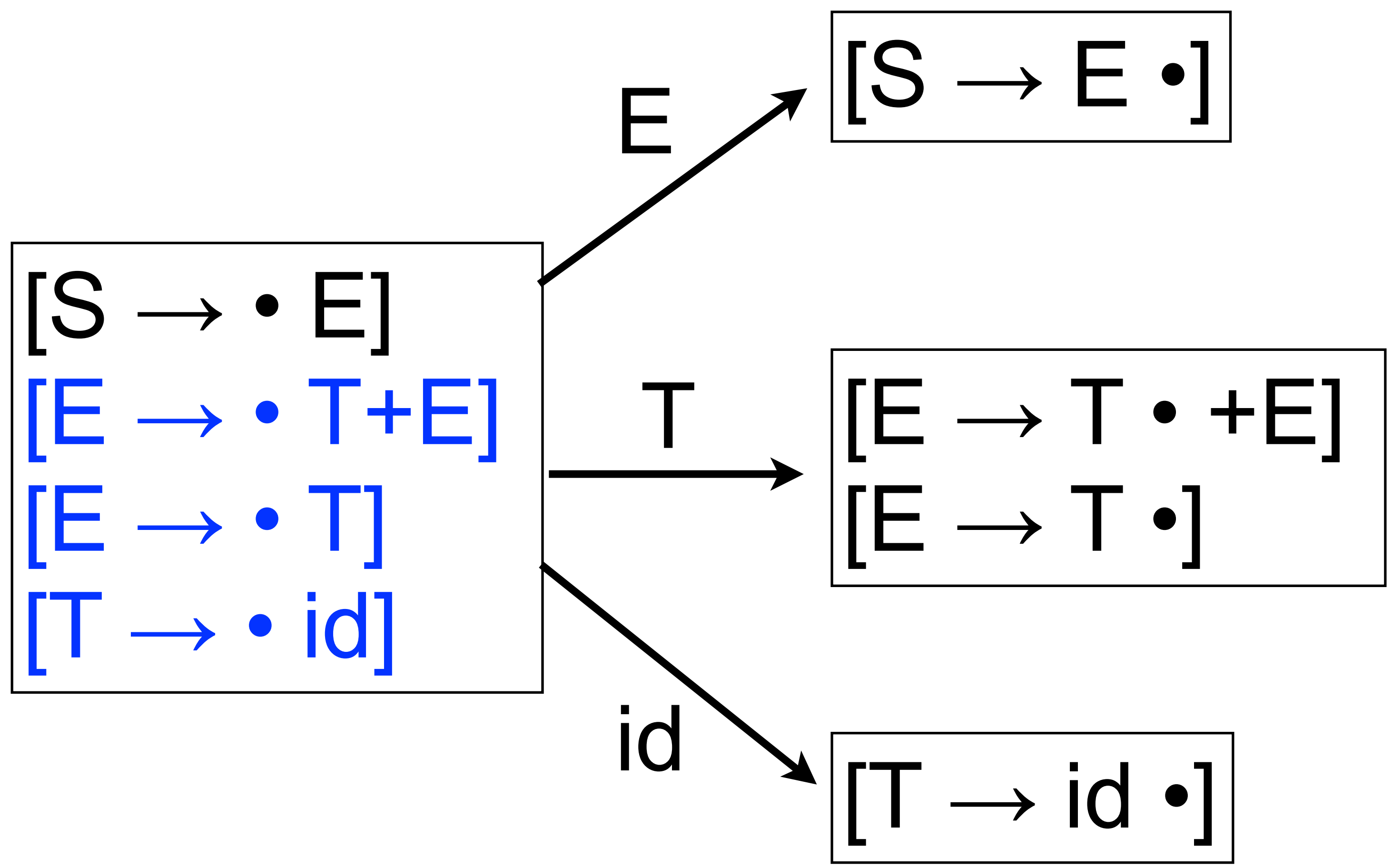
Example — goto with LR(0)

$S \rightarrow E$

$E \rightarrow T+E$

$\quad | \;\; T$

$T \rightarrow id$



[kernel item]
[derived item]

Example — goto with LR(1)

$S \rightarrow E$

$E \rightarrow T+E$

$\quad\mid\ T$

$T \rightarrow id$

[kernel item]
[derived item]

[S → • E, $]
[E → • T+E, $]
[E → • T, $]
[T → • id, +]
[T → • id, $]

E

[S → E •, $]

T

[E → T • +E, $]
[E → T •, $]

id

[T → id •, +]
[T → id •, $]

Building parser states

$cc_0 \leftarrow$ closure ([S'$\rightarrow \bullet$S, $\underline{\$}$])
$CC \leftarrow \{ cc_0 \}$

while (new sets are still being added to CC)
  for each unmarked set $cc_j \in CC$
    mark cc_j as processed
    for each x following a $\bullet$ in an item in cc_j
      temp $\leftarrow$ goto(cc_j, x)
      if temp $\notin$ CC
        then $CC \leftarrow CC \cup \{ temp \}$
      record transitions from cc_j to temp on x

- CC = canonical collection (of LR(k) items)
- Fixpoint computation (worklist version)
- Loop adds to CC
  - $CC \subseteq 2^{ITEMS}$, so CC is finite

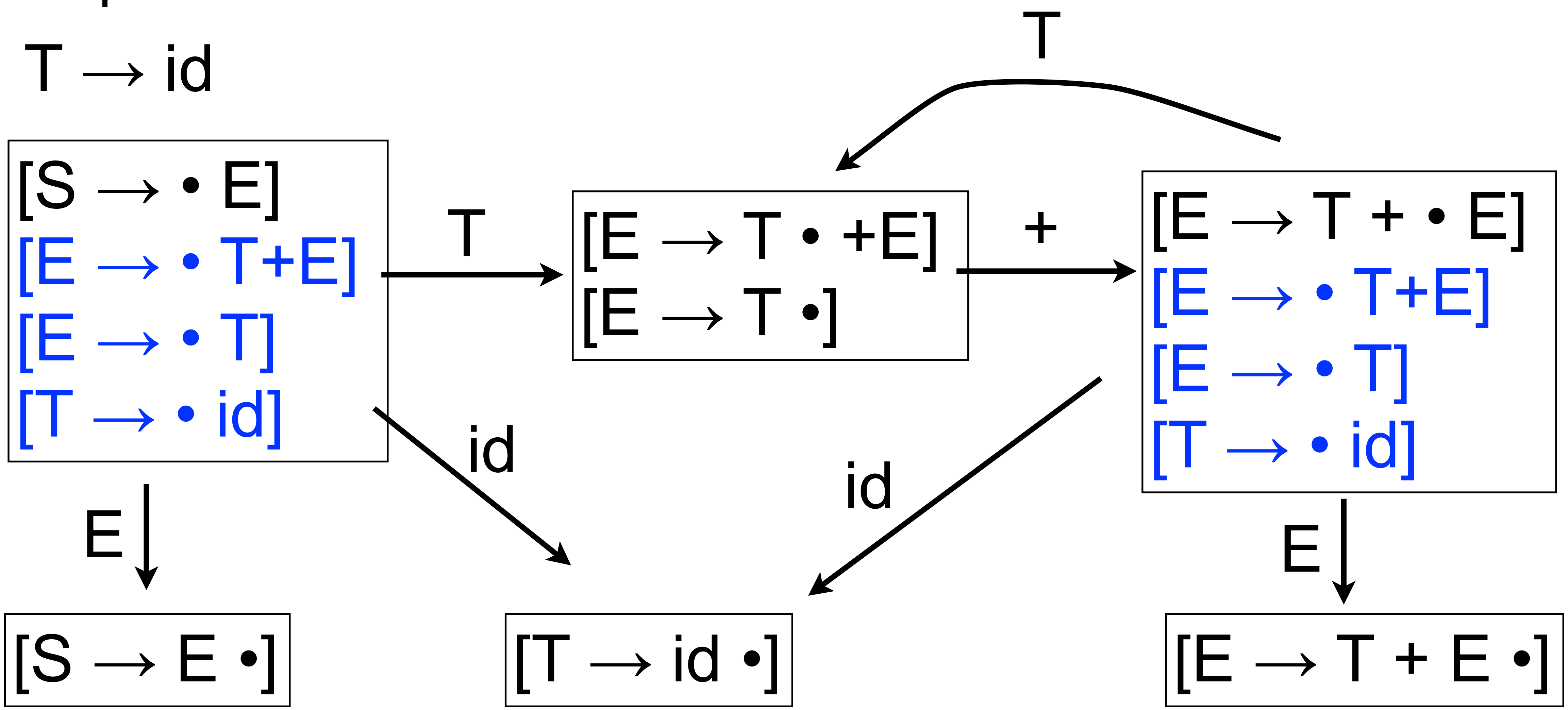
Example LR(0) states

$S \rightarrow E$

$E \rightarrow T+E$

$\quad | \ T$

$T \rightarrow id$



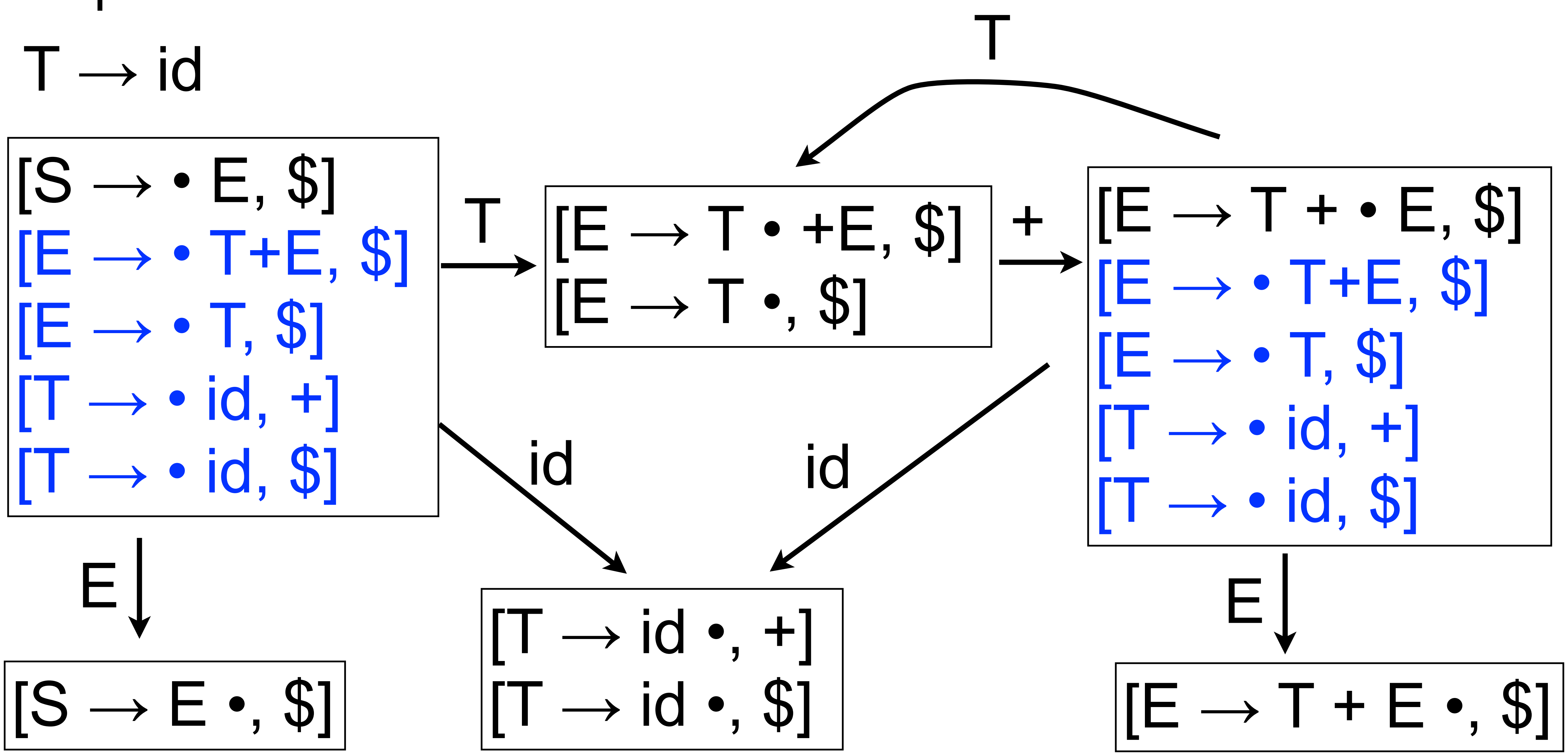
Example LR(1) states

$S \rightarrow E$

$E \rightarrow T{+}E$

$\quad \mid \ T$

$T \rightarrow id$



Building ACTION and GOTO tables

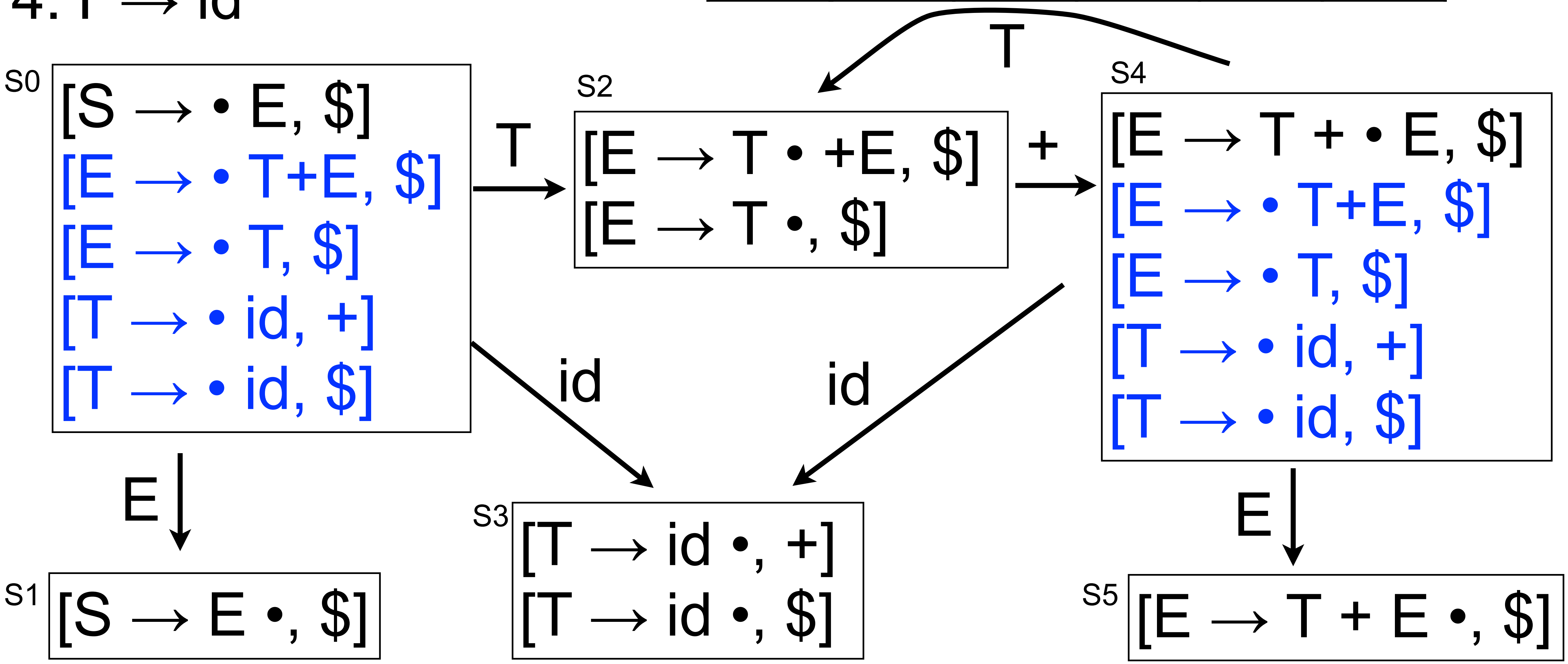
```
∀ set s_x ∈ S
   ∀ item i ∈ s_x
      if i is [A→β •a γ,b] and goto(s_x,a) = s_k, a ∈ terminals  // • to left of terminal a
          then ACTION[x,a] ← "shift k"                           // ⇒ shift if lookahead = a

      else if i is [S'→S •,$]                                    // start production done,
          then ACTION[x , $] ← "accept"                          // ⇒ accept if lookahead = $

      else if i is [A→β •,a]                                     // • all the way to right
          then ACTION[x,a] ← "reduce A→β"                        // → production done
   ∀ n ∈ nonterminals                                            // reduce if lookahead = a
      if goto(s_x ,n) = s_k
          then GOTO[x,n] ← k                                     // store transitions for nonterminals
```

- Many items generate no table entry
  - e.g., [A→β·Bα,a] does not, but closure ensures that all the rhs's for B are in sx

Ex ACTION and GOTO tables

1. S → E
2. E → T+E
3. | T
4. T → id

	ACTION			GOTO	
	id	+	$	E	T
S0	s3			1	2
S1			acc		
S2		s4	r3		
S3		r4	r4		
S4	s3			5	2
S5			r2		

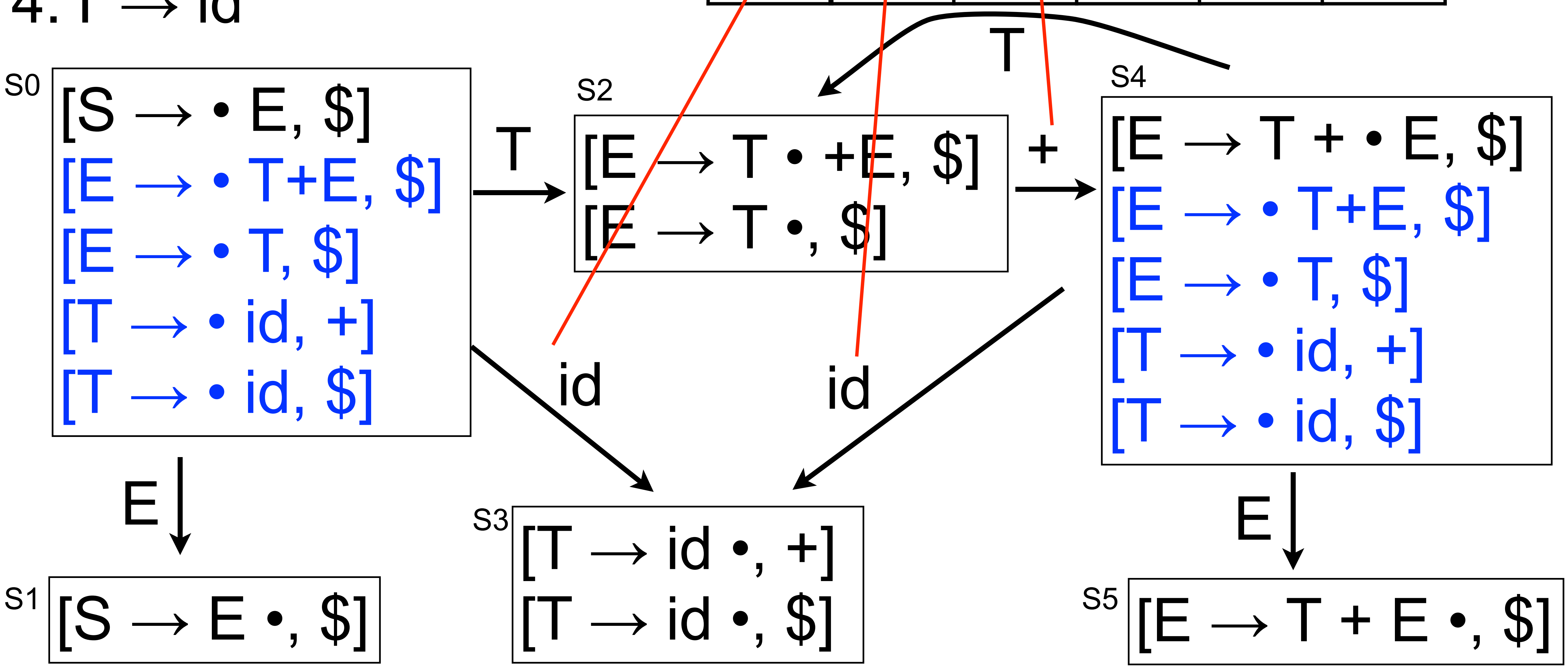


Ex ACTION and GOTO tables

1. S → E
2. E → T+E
3. | T
4. T → id

Entries for shift

	ACTION			GOTO	
	id	+	$	E	T
S0	s3			1	2
S1			acc		
S2		s4	r3		
S3		r4	r4		
S4	s3			5	2
S5			r2		

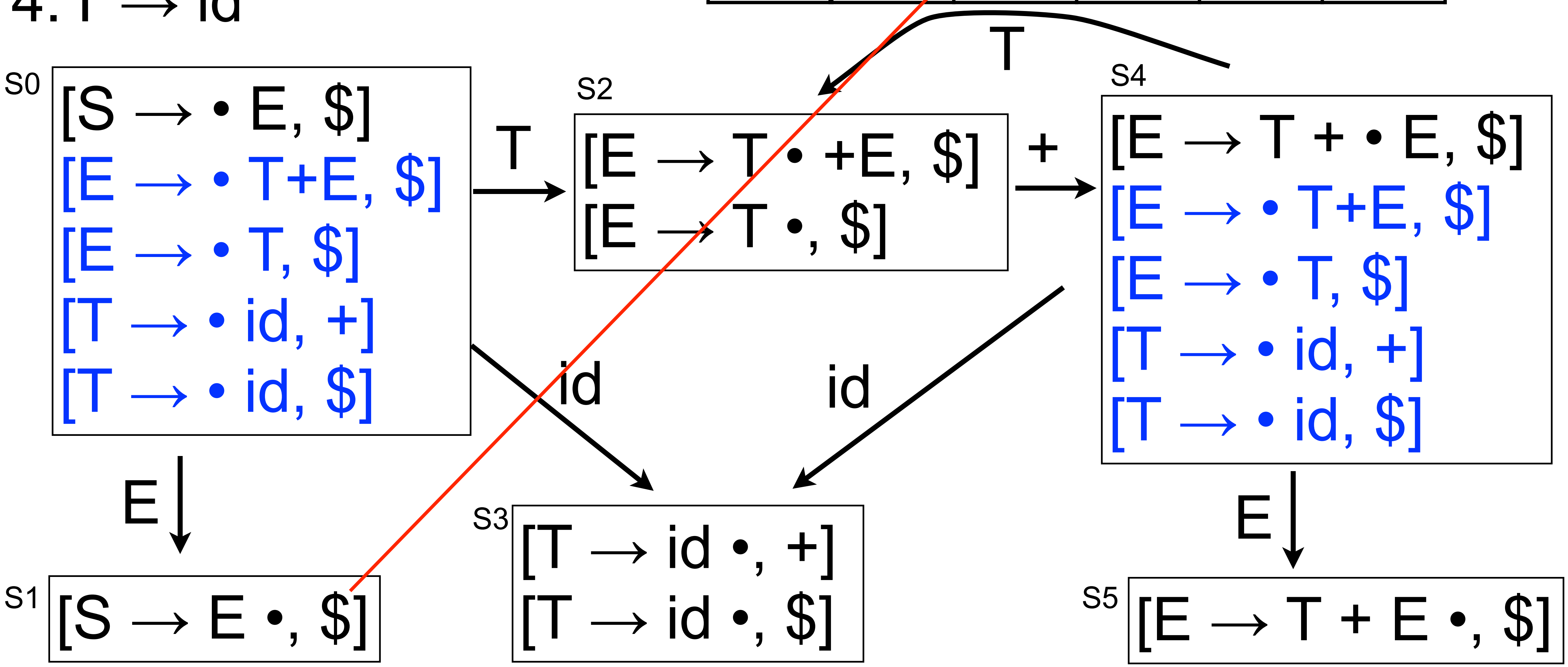


Ex ACTION and GOTO tables

1. S → E
2. E → T+E
3. | T
4. T → id

Entry for accept

	ACTION			GOTO	
	id	+	$	E	T
S0	s3			1	2
S1			acc		
S2		s4	r3		
S3		r4	r4		
S4	s3			5	2
S5			r2		

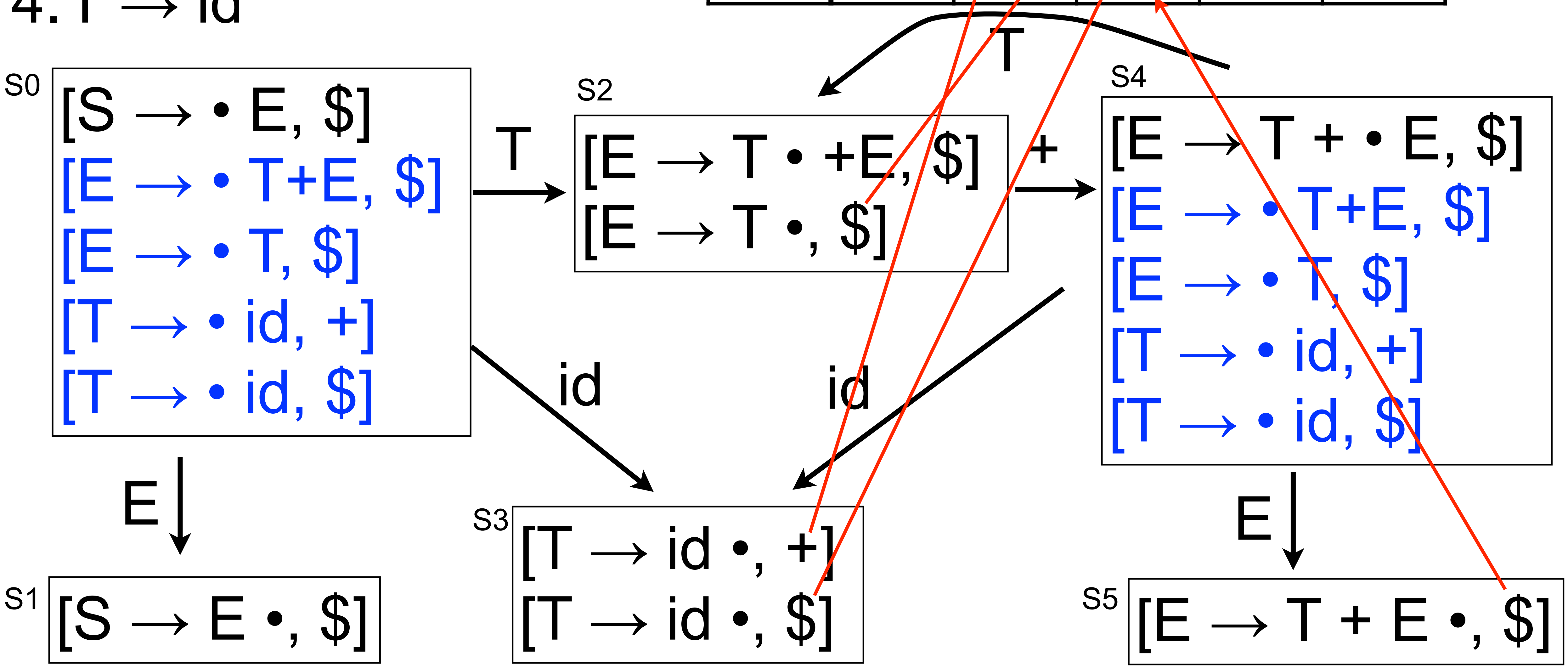


Ex ACTION and GOTO tables

1. S → E
2. E → T+E
3. | T
4. T → id

Entries for reduce

	ACTION			GOTO	
	id	+	$	E	T
S0	s3			1	2
S1			acc		
S2		s4	r3		
S3		r4	r4		
S4	s3			5	2
S5			r2		

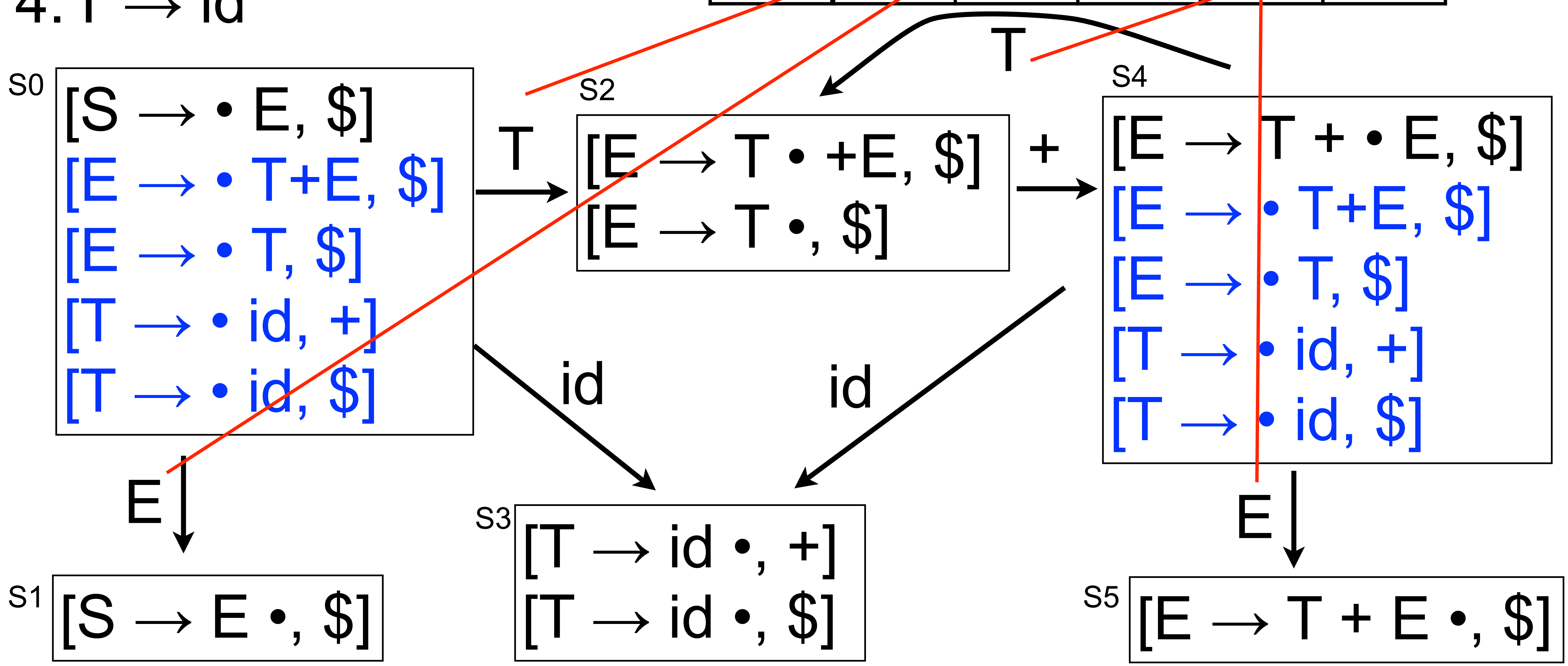


Ex ACTION and GOTO tables

1. $S \rightarrow E$
2. $E \rightarrow T+E$
3. $\quad | \;\; T$
4. $T \rightarrow id$

Entries for GOTO

	ACTION			GOTO	
	id	+	$	E	T
S0	s3			1	2
S1			acc		
S2		s4	r3		
S3		r4	r4		
S4	s3			5	2
S5			r2		



What can go wrong?

- What if set s contains $[A \rightarrow \beta \bullet a\gamma, b]$ and $[B \rightarrow \beta \bullet, a]$?
  - First item generates "shift", second generates "reduce"
  - Both define ACTION[s,a] — cannot do both actions
  - This is a *shift/reduce conflict*
- What if set s contains $[A \rightarrow \gamma \bullet, a]$ and $[B \rightarrow \gamma \bullet, a]$?
  - Each generates "reduce", but with a different production
  - Both define ACTION[s,a] — cannot do both reductions
  - This is called a reduce/reduce conflict
- In either case, the grammar is not LR(1)

Shift/reduce conflict

```
%token <int> INT
%token EOL PLUS LPAREN RPAREN
%start main                /* the entry point */
%type <int> main
%%
main:
| expr EOL                 { $1 }
expr:
| INT                      { $1 }
| expr PLUS expr           { $1 + $3 }
| LPAREN expr RPAREN       { $2 }
```

- Associativity unspecified
  - Ambiguous grammars always have conflicts
  - But, some non-ambiguous grammars also have conflicts

Solving conflicts

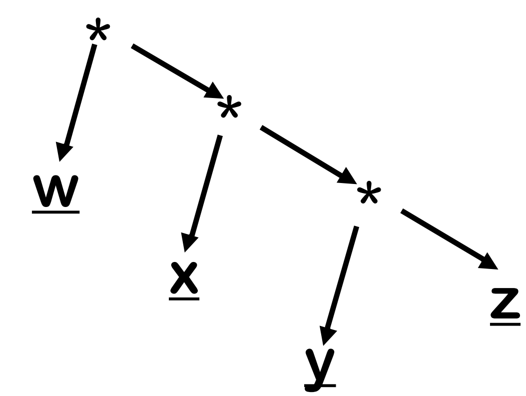
- Refactor grammar
- Specify operator precedence and associativity

```
%left PLUS MINUS          /* lowest precedence */
%left TIMES DIV           /* medium precedence */
%nonassoc UMINUS          /* highest precedence */
```

  - Lots of details here
    - See “12.4.2 Declarations” at
    - http://caml.inria.fr/pub/docs/manual-ocaml/manual026.html#htoc151
  - When comparing operator on stack with lookahead
    - Shift if lookahead has higher prec OR same prec, right assoc
    - Reduce if lookahead has lower prec OR same prec, left assoc
  - Can use smaller, simpler (ambiguous) grammars
    - Like the one we just saw

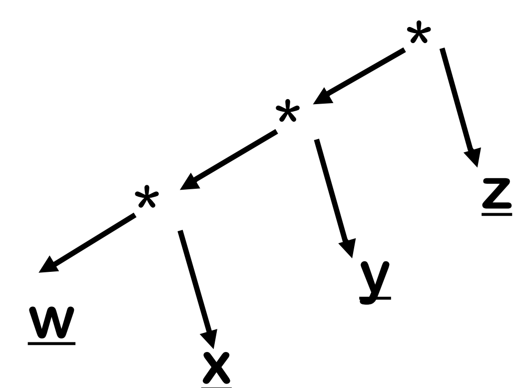
Left vs. right recursion

- Right recursion
  - Required for termination in top-down parsers
  - Produces right-associative operators



w * (x * (y * z))

- Left recursion
  - Works fine in bottom-up parsers
  - Limits required stack space
  - Produces left-associative operators



((w * x) * y) * z

- Rule of thumb
  - Left recursion for bottom-up parsers
  - Right recursion for top-down parsers

Reduce/reduce conflict (1)

```
%token <int> INT
%token EOL PLUS LPAREN RPAREN
%start main                /* the entry point */
%type <int> main
%%
main:
| expr EOL                 { $1 }
expr:
| INT                      { $1 }
| term                     { $1 }
| term PLUS expr           { $1 + $3 }
term :
| INT                      { $1 }
| LPAREN expr RPAREN       { $2 }
```

- Often these conflicts suggest a serious problem
  - Here, there’s a deep amigiuity

Reduce/reduce conflict (2)

```
%token <int> INT
%token EOL PLUS LPAREN RPAREN
%start main                   /* the entry point */
%type <int> main
%%
main:
| expr EOL                    { $1 }
expr:
| term1                       { $1 }
| term1 PLUS PLUS expr        { $1 + $4 }
| term2 PLUS expr             { $1 + $3 }
term1 :
| INT                         { $1 }
| LPAREN expr RPAREN          { $2 }
term2 :
| INT                         { $1 }
```

- Grammar not ambiguous, but not enough lookahead to distinguish last two expr productions

Shrinking the tables

- Combine terminals
  - E.g., number and identifier, or + and -, or * and /
    - Directly removes a column, may remove a row
- Combine rows or columns (*table compression*)
  - Implement identical rows once and remap states
  - Requires extra indirection on each lookup
  - Use separate mapping for ACTION and for GOTO
- Use another construction algorithm
  - LALR(1) used by ocamlyacc

LALR(1) parser

- Define the *core* of a set of LR(1) items as
  - Set of LR(0) items derived by ignoring lookahead symbols

LR(1) state	Core
$[E \rightarrow a\ \bullet, b]$ $[A \rightarrow a\ \bullet, c]$	$[E \rightarrow a\ \bullet]$ $[A \rightarrow a\ \bullet]$

- LALR(1) parser merges two states if they have the same core
- Result
  - Potentially much smaller set of states
  - May introduce reduce/reduce conflicts
  - Will not introduce shift/reduce conflicts

LALR(1) example

[E → a •, b]
[A → ba •, c]

[E → a •, d]
[A → ba •, b]

LR(1) states

[E → a •, b]
[A → ba •, c]
[E → a •, d]
[A → ba •, b]

Merged state

- Introduces reduce/reduce conflict
  - Can reduce either E → a or A → ba for lookahead = b

LALR(1) vs. LR(1)

- Example grammar

$$
\begin{aligned}
&S' \rightarrow S \\
&S \rightarrow aAd \mid bBd \mid aBe \mid bAe \\
&A \rightarrow c \\
&B \rightarrow c
\end{aligned}
$$

- LR(0) ?
- LR(1) ?
- LALR(1) ?

LR(k) Parsers

- Properties
  - Strictly more powerful than LL(k) parsers
  - Most general non-backtracking shift-reduce parser
  - Detects error as soon as possible in left-to-right scan of input
    - Contents of stack are viable prefixes
      - Possible for remaining input to lead to successful parse

Error handling (lexing)

- What happens when input not handled by any lexing rule?
  - An exception gets raised
  - Better to provide more information, e.g.,

```
rule token = parse
...

| _ as lxm { Printf.printf "Illegal character %c" lxm;
             failwith "Bad input" }
```

- Even better, keep track of line numbers
  - Store in a global-ish variable (oh no!)
  - Increment as a side effect whenever \n recognized

Error handling (parsing)

- What happens when parsing a string not in the grammar?
  - Reject the input
  - Do we keep going, parsing more characters?
    - May cause a cascade of error messages
    - Could be more useful to programmer, if they don't need to stop at the first error message (what do you do, in practice?)

- Ocamlyacc includes a basic error recovery mechanism
  - Special token error may appear in rhs of production
  - Matches erroneous input, allowing recovery

Error example (1)

```
...
expr:
| term                  { $1 }
| expr PLUS term        { $1 + $3 }
| error            { Printf.printf "invalid expression"; 0 }
term: ...
```

- If unexpected input appears while trying to match expr, match token to error
  - Effectively treats token as if it is produced from expr
  - Triggers error action

Error example (2)

```
...
term:
| INT                    { $1 }
| LPAREN expr RPAREN     { $2 }
| LPAREN error RPAREN {Printf.printf "Syntax error!\n"; 0}
```

- If unexpected input appears while trying to match term, match tokens to error
  - Pop every state off the stack until LPAREN on top
  - Scan tokens up to RPAREN, and discard those, also
  - Then match error production

Error recovery in practice

- A very hard thing to get right!
  - Necessarily involves guessing at what malformed inputs you may see
- How useful is recovery?
  - Compilers are very fast today, so not so bad to stop at first error message, fix it, and go on
  - On the other hand, that does involve some delay
- Perhaps the most important feature is *good error messages*
  - Error recovery features useful for this, as well
  - Some compilers are better at this than others

OCamlyacc tip

- Setting OCAMLRUNPARAM=p will cause the parsing steps to be printed out as the parser runs
- (And setting OCAMLRUNPARAM=b will tell OCaml to print a stack backtrace for any thrown exceptions.)

Real programming languages

- Essentially all real programming languages don't quite work with parser generators
  - Even Java is not quite LALR(1)

- Thus, real implementations play tricks with parsing actions to resolve conflicts

- In-class exercise: C typedefs and identifier declarations/definitions

Additional Parsing Technologies

- For a long time, parsing was a “dead” field
  - Considered solved a long time ago
- Recently, people have come back to it
  - LALR parsing can have unnecessary parsing conflicts
  - LALR parsing tradeoffs more important when computers were slower and memory was smaller
- Many recent new (or new-old) parsing techniques
  - GLR — generalized LR parsing, for ambiguous grammars
  - LL(*) — ANTLR
  - Packrat parsing — for *parsing expression grammars*
  - etc...
- The input syntax to many of these looks like yacc/ lex

Designing language syntax

- Idea 1: Make it look like other, popular languages
  - Java did this (OO with C syntax)
- Idea 2: Make it look like the domain
  - There may be well-established notation in the domain (e.g., mathematics)
  - Domain experts already know that notation
- Idea 3: Measure design choices
  - E.g., ask users to perform programming (or related) task with various choices of syntax, evaluate performance, survey them on understanding
    - This is very hard to do!
- Idea 4: Make your users adapt
  - People are really good at learning...