1. Give regular expressions for the following languages.
   (a) \( \{ w \in \{a, b\}^*: \text{exactly one } a \} \).
   (b) \( \{ w \in \{a, b\}^*: \text{exactly one } a \text{ or one } b \} \).
   (c) \( \{ w \in \{a, b\}^*: \text{begins with } aa \text{ and ends with } bb \} \).
   (d) \( \{ w \in \{a, b\}^*: \text{begins with } ab \text{ and ends with } ba \} \).
   (e) \( \{ w \in \{a, b\}^*: \text{does not begin with } ab \} \).
   (f) \( \{ w \in \{a, b\}^*: \text{both } aabb \text{ and } bbaa \text{ are substrings} \} \).
   (g) \( \{ w \in \{a, b\}^*: \text{either } aabb \text{ or } bbaa \text{ is a substring (or both)} \} \).

2. Prove that the following languages over the alphabet \( \Sigma = \{a, b\} \) are not regular.
   (a) \( L = \{ a^n ba^{3n} \mid n > 0 \} \)
   (b) \( L = \{ a^n b^n a^n \mid n > 0 \} \)
   (c) \( L = \{ a^n b^i \mid n > 0, i = n \text{ or } i = 2n \} \)
   (d) \( L = \{ ww^R \mid \text{where } w^R \text{ is the reverse of } w \} \)

3. Consider the following languages over the alphabet \( \Sigma = \{a, b\} \). Answer true or false to the following statements. Justify your answers. Note that if a statement is false, the simplest proof is usually but not always a counterexample.
   (a) If \( L_1 \) is nonregular and \( L_1 \subset L_2 \) then \( L_2 \) is nonregular.
   (b) If \( L_2 \) is nonregular and \( L_1 \subset L_2 \) then \( L_1 \) is nonregular.
   (c) If \( L \) is nonregular then its complement \( \overline{L} \) is nonregular.
   (d) If \( L_1 \) is regular, then \( L_1 \cup L_2 \) is regular for any language \( L_2 \).
   (e) If \( L_1 \) and \( L_2 \) are nonregular, then \( L_1 \cap L_2 \) is nonregular.

4. Consider the following regular expression:
   \[ a^* b^* (aa \cup bbb)^* \]
   Convert this regular expression into a NFA using the construction given in class.

5. Consider the following language over the alphabet \( \Sigma = \{a, b\} \).
   \( L = \{ w \mid \text{all } a's \text{ in } w \text{ come before all } b's \text{ and } |w| \geq 1 \} \)
   (a) Give a DFA with at most four states that accepts \( L \).
   (b) Convert this DFA into a regular expression using the construction given in class.
   (c) Simplify your final regular expression.