1. (a) Give an example of a convex function that is not proper.

(b) Give an example of a convex function that is bounded below, but has no minimizer.

(c) Give an example of a convex function that is not closed. Hint: Suppose I gave you a convex function that holds beer. What could you do to make all the beer leak out?

(d) Give an example of a convex function that has unbounded level sets, but has at least one minimizer.
2. For this problem, you may use any of the results/rules from the lecture slides.

(a) Consider the set of probability functions \( \{ f | f \geq 0, \text{ and } \int f = 1 \} \). Is this a convex set?

(b) Consider the function \( M_5 : \mathbb{R}^{10} \rightarrow \mathbb{R} \), where \( M_5(z) \) returns the sum of the 5 largest entries in \( x \). Is this function convex? Why?

(c) Consider the set of low-rank matrices \( \{ A | \text{rank}(A) < k \} \). Is this set convex? Why?
(d) Let $C \in \mathbb{R}^n$ be a convex set. For a scalar $x$, let $C(x)$ be the set of all points in $C$ whose first coordinate is closest to $x$. Is $C(x)$ a convex set for any choice of $C$ and $x$? Why?

(e) Is the set $C = \{(x, y) \mid \|x\| \leq y\}$ convex? Is it a cone? Why? What about the set $C = \{(x, y) \mid \|x\|^2 \leq y\}$? Is this a convex cone?
3. Suppose \( f \) is strongly convex. Show that the gradient of \( f \) is strongly monotone, i.e., there is a constant \( m > 0 \) with
\[
\langle x - y, \nabla f(x) - \nabla f(y) \rangle \geq m\|x - y\|^2.
\]

4. Suppose that \( g(x) \) is convex and \( h(x) \) is concave (i.e. \(-h(x)\) is convex). Suppose we restrict both functions into a closed, convex set \( C \) such that both \( g(x) \) and \( h(x) \) are always positive when \( x \in C \). Prove that the function \( f(x) = g(x)/h(x) \) is quasi-convex, and therefore every local minimum is also global.