1. Consider the “monotropic” program

\[
\begin{align*}
\text{minimize} & \quad \|x\|_\infty \\
\text{subject to} & \quad Ax = b.
\end{align*}
\]

(a) Write this as an unconstrained (or implicitly constrained) problem using the characteristic function of the zero vector \(\chi_0(z)\). This function is zero if it’s argument is zero, and infinite otherwise.

**Solution:** Put your solution here

(b) What is the conjugate of \(f(z) = \|z\|_\infty\)?

**Solution:** Put your solution here

(c) What is the conjugate of \(g(z) = \chi_0(z)\)?

**Solution:** Put your solution here

(d) Using the conjugate functions, write down the dual of (1).

**Solution:** Put your solution here

2. Consider the linear program

\[
\begin{align*}
\text{minimize} & \quad c^T x \\
\text{subject to} & \quad Ax = b \\
& \quad x \geq 0.
\end{align*}
\]

(a) Write the optimality conditions for this problem (i.e., the KKT system).

**Solution:** Put your solution here

(b) Write the Lagrangian for this problem.

**Solution:** Put your solution here

(c) Minimize out the primal variables in the Lagrangian, and write the dual formulation of this linear program.
3. Consider the problem

\[ \begin{align*}
\text{minimize} & \quad f(x) \\
\text{subject to} & \quad g(x) \leq 0.
\end{align*} \]

Let \( x_0 \) be a solution to this problem, and \( \lambda_0 \) be the corresponding optimal Lagrange multiplier. Now, define a perturbed problem

\[ \begin{align*}
\text{minimize} & \quad f(x) \\
\text{subject to} & \quad g(x) \leq \epsilon
\end{align*} \]

where \( \epsilon \) is a vector. Let \( x_\epsilon \) be a solution to the perturbed problem. Note, if we put large negative values in \( \epsilon \), then the constraint set gets smaller, and we expect the corresponding value of \( f(x_\epsilon) \) to increase. Prove the “sensitivity bound”

\[ f(x_0) - \lambda_0^T \epsilon \leq f(x_\epsilon). \]

This bound shows that the Lagrange multipliers determine how much the objective increases as the vector \( \epsilon \) becomes more negative.

**Solution:** Put your solution here