

Closed book. Closed notes. No electronic device.

1.

For each description below, give the term ( $\leq 4$  words) that *best* describes it. In most cases, the term will be in the table at left. In a few cases, it will not.

AES
Authenticity
Block cipher
Collision resistant
CA
Certificate
Certificate chain
CRL
Confidentiality
DES
Diffie-Helman
Dictionary attack
Euclid's algorithm
Eucler's theorem
Existential forgery
HMAC
Integrity
KDC
MAC
Mode
CBC
CTR
OFB
Non-repudiation
OCSP
OCSP stapling
One-time pad
Pre-image resistant
PGP
PKCS
PKI
RSA
Session key
Signing
Ticket
Totient function
Verification

- This ensures that the data can be read only by the intended receiver.  
**Solution:** Confidentiality
- This ensures that any modification to the data is detected by the intended receiver.  
**Solution:** Integrity
- This ensures that data received was sent by the specified sender.  
**Solution:** Authenticity
- This ensures that a third party can verify that the data was sent by the specified sender. **Solution:** Non-repudiation
- This kind of crypto uses different keys for encryption and decryption.  
**Solution:** Asymmetric crypto. (Alternative: Public-key crypto)
- The attack model in which the attacker has access to an encryption oracle but not a decryption oracle. **Solution:** Chosen plaintext attack
- This symmetric block cipher supports only one key size. **Solution:** DES
- This symmetric block cipher supports multiple key sizes. **Solution:** AES
- This defines how to use a block cipher on arbitrary-size data. **Solution:** Mode
- This ensures that encrypting the same message more than once results in different ciphertext. **Solution:** Initialization vector (IV)
- This mode allows the block cipher encryption function calls to be made before the data is available. **Solution:** OFB (Alternative: CTR)
- This mode allows encryption to be done in parallel.  
**Solution:** OFB (Alternative: CTR)
- This mode allows a hash function to be used for encryption of arbitrary-size data.  
**Solution:** OFB (Alternative: CTR)
- The property of a hash function that makes it hard to find a message  $m$  that hashes to a given number. **Solution:** Pre-image resistant
- This is the standard method for generating MACs from block ciphers.  
**Solution:** ECBC (Encrypted Cipher Block Chaining)
- This is the standard method for generating MACs from hash functions.  
**Solution:** HMAC (Hashed MAC)
- This is the set of integers in  $1, \dots, n - 1$  that are relatively prime to  $n$ .  
**Solution:**  $Z_n^*$
- This is the number of integers in  $1, \dots, n - 1$  that are relatively prime to  $n$ .  
**Solution:**  $\phi(n)$  (totient of  $n$ )
- What do we need to efficiently compute the number of integers in  $1, \dots, n - 1$  that are relatively prime to  $n$ . **Solution:** Prime factors of  $n$
- What allows us to efficiently compute  $\phi(n)$ . **Solution:** Prime factors of  $n$
- An attack that goes through a set of candidate passwords.  
**Solution:** Dictionary attack
- This means that after a session-key is forgotten by the principals that used it, no one can decrypt data encrypted with that key. **Solution:** Perfect-forward secrecy
- A public-key infrastructure that is not hierchical. **Solution:** PGP

2. Alice has an account with a server. The server makes her change her password every few months, to which Alice just increments a number in her password, e.g., `pwd1`, `pwd2`, `...`.

Why does the server not complain that the new password is very much like her old one?

**Solution** Because the server does not have the old password (only a hash of it).

3. Let  $[e, n]$  be the RSA public key of a server. Suppose someone gives you the prime factors of  $n$ , say  $p$  and  $q$ . Can you obtain the private key  $[d, n]$ ? If not, explain briefly. If yes, briefly give the steps.

**Solution**

Yes

$$\phi \leftarrow (p-1) \cdot (q-1)$$

$$d \leftarrow e^{-1} \bmod \phi \text{ (using Euclid's algorithm).}$$

4. A hash function  $H()$  generates a 256-bit hash. How many random messages on average would one have to hash before finding two distinct messages that hash to the same value.

**Solution:** Of the order of  $\sqrt{2^{256}} (= 2^{128})$  messages

5. Is a strong password significantly better than a weak password against an online dictionary attack. Explain briefly.

**Solution:** No. Because failed attempts are limited in number/frequency

6. Is a strong password significantly better than a weak password against an offline dictionary attack. Explain briefly.

**Solution:** Yes, especially if the attacker wants any password out of a large set. A weak password will be cracked before a strong password.

7. A server has  $N$  users and stores hashes of the users' passwords in a map  $P$  indexed by user id. Specifically, for user  $u$ , the entry  $P(u)$  is  $H^4(p)$ , where  $p$  is  $u$ 's password,  $H$  is a hash function, and  $H^4(p)$  is  $H(H(H(H(p))))$ .

a. What would the entry be if the entries were also "salted".

**Solution:**  $P(u)$  is  $[salt, H^4(salt||p)]$

b. If  $N$  is 20, does salting improve security significantly? Explain briefly.

**Solution:** No. It's unlikely that the same password would be used in a list of 20 users.

8. Let  $x$  be an element of  $Z_n$  and  $y$  denote its multiplicative-inverse-mod- $n$ .

a. When does  $y$  exist?

**Solution:** Iff  $\gcd(x, n) = 1$

b. Give the equation that  $x$  and  $y$  satisfy.

**Solution:**  $(x \cdot y) \bmod n = 1$

9. Let  $E(k, \cdot)$  and  $D(k, \cdot)$  denote AES encryption and decryption using key  $k$ . Let message  $msg$  consist of blocks  $[m_1, \dots, m_n]$ . Let  $[c_0, c_1, \dots, c_n]$  be the ciphertext resulting from encrypting  $msg$  using AES with key  $k$  in some mode.

a. Assume CBC mode. Express  $c_i$  in terms of  $E()$  and  $D()$  and any arguments, for  $i = 1, \dots, n$ .

**Solution:**

$$c_0 = \text{random IV}$$

$$c_i = E(k, m_i \oplus c_{i-1}) \text{ for } i = 1, \dots, n$$

b. Assume CTR mode. Express  $c_i$  in terms of  $E()$  and  $D()$  and any arguments, for  $i = 1, \dots, n$ .

**Solution:**

$$c_0 = \text{random IV}$$

$$c_i = E(k, c_0 + i) \oplus m_1 \text{ for } i = 1, \dots, n$$

10. For an arbitrary-size message  $msg$ , let  $M(k, msg)$  denote the last block of CBC-AES encryption using key  $k$  and IV=0. Is  $M(k, msg)$  a secure MAC. If you answer yes, explain briefly. If you answer no, give a counter example.

**Solution:**

No.

It is vulnerable to existential forgery.

1. Create message  $msg$ . Get its mac  $t (= M(k, msg))$ .
2. Create single-block message  $m$ .
3. Create single-block message  $m \oplus t$ . Get its mac  $t' (= M(k, m \oplus t))$ .

$t'$  is a valid tag for message  $msg||m$ . (The ciphertext for block  $m$  is  $E(k, x \oplus m)$ , where  $x$  is the ciphertext for the block preceding  $m$ . But  $x$  is the same as  $t$ .)

11. Why is a random pad needed for RSA encryption of a msg.

**Solution:**

Two reasons:

- So that repeated encryptions of the same msg yield different ciphertexts.
- So that scrambling happens even if the message is small and  $e$  is small.

**12.** Server  $B$  has a well-known fixed IP address and TCP port, and no other service can use that address and port. User  $A$  shares a password, say  $pwd$ , with  $B$ .  $A$  connects as follows:

1. Establish a shared key  $s$  with a standard (not authenticated) Diffie-Helman.
  2. Send [ $A$ ,  $pwd$ ] encrypted with key  $s$  to the server.
  3.  $B$  authenticates the user if the password matches.
- a. Assume an attacker that can only eavesdrop on messages. Does the above ensure that key  $s$  is securely shared between  $A$  and  $B$ . If you answer “no”, give an attack. If you answer “yes”, explain.

**Solution:**

Yes.

When  $A$  establishes a TCP connection to  $B$ 's IP address and TCP port,  $A$  is assured it is talking to  $B$  (because no one else can use that address and port, and the attacker cannot tamper with messages). So after step 1,  $A$  is assured that the DH key  $s$  is shared with  $B$ .

After step 1,  $B$  is assured it has a DH key  $s$  with someone (need not be  $A$ ). But after step 2,  $B$  is assured that the DH key is shared with  $A$ .

- b. Repeat part a for an attacker that can eavesdrop and tamper with messages (intercept and change them).

**Solution:**

No.

The classic MITM (man-in-the-middle attack) works here.

- a1.  $A$  establishes a TCP connection to  $B$ .
- a2.  $A$  generates random  $x$  and sends  $g^x \text{ mod } p$ .
- a3. Attacker intercepts this and does the following:
  - \* generate random  $y$
  - \* set DH key, say  $t_A \leftarrow g^{x \cdot y} \text{ mod } p$
  - \* send  $g^y \text{ mod } p$  to  $B$
- a4. When  $B$  receives the attacker's  $g^y \text{ mod } p$  (from a3),  $B$  generates random  $z$ , sets DH key, say  $s_B \leftarrow g^{z \cdot y} \text{ mod } p$ , and sends  $g^z \text{ mod } p$ .
- a5. Attacker intercepts this and does the following:
  - \* set DH key, say  $t_B \leftarrow g^{z \cdot y} \text{ mod } p$
  - \* send  $g^y \text{ mod } p$  to  $A$
- a6. When  $A$  receives the attacker's  $g^y \text{ mod } p$  (from a5), it sets DH key, say  $s_A \leftarrow g^{x \cdot y} \text{ mod } p$ .

**[At this point,  $A$  and attacker share DH key  $s_A$ , and  $B$  and attacker share DH key  $s_B$ .]**

- a7.  $A$  sends [ $A$ ,  $pwd$ ] encrypted with key  $s_A$ .
- a8. Attacker intercepts this and does the following:
  - \* decrypts it (using  $s_A$ )
  - \* encrypts it using  $s_B$  and sends it to  $B$ .

**Attacker now has  $pwd$ .**

- a9.  $B$  receives attacker's msg (from a7), verifies  $pwd$ , and now treats DH key  $s_B$  as shared with  $A$  (whereas it is actually shared with the attacker).

**13.** A domain has a CA  $X$ , which is the trust anchor for the domain's users.

- a. What steps does a new user, say  $A$ , take upon joining the domain.

**Solution:**

$A$  generates a new public-key pair, say  $[sk_A, pk_A]$ .

$A$  gets from  $X$  a certificate for  $A$ 's public key, say  $cert_{X,A}$ .

$A$  gets  $X$ 's public key.

- b. What steps are taken when a user, say  $A$ , leaves the domain before its certificate expires.

**Solution:**  $X$  adds the certificate's serial number to the next CRL it issues, and gives a "not valid" response to any OCSP query for the certificate.

- c. What steps are taken when  $X$ 's secret key is exposed.

**Solution:**

$X$  generates a new public-key pair

$X$  issues a new certificate (using the new key) for every  $A_i$

Every  $A_i$  deletes its old public key of  $X$

Every  $A_i$  gets (securely) the new public key of  $X$

**Note**

- $X$  issuing a CRL using the old key is useless. (Could be issued by the attacker).
- $X$  issuing a CRL using the new key is useless. (After  $A_i$  deletes  $X$ 's old pub key, the old certs won't work.)

**14.** The users in domain  $x.com$  has a CA  $X$  as trust anchor. The users in domain  $y.com$  has a CA  $Y$  as trust anchor. One day,  $x.com$  and  $y.com$  are acquired by  $z.com$ , which has a CA  $Z$  as trust anchor.

List the steps that will allow users in all three domains to talk to each other. Minimize the number of new certificates that are issued.

**Solution:**

1.  $Z$  issues certificates for  $X$  and  $Y$ .
2. Users in  $x.com$  and  $y.com$  get  $Z$ 's public key and add  $Z$  as a trust anchor.

**15.** A domain's authentication is handled by KDC  $X$ .

- a. What steps does a new user, say  $A$ , take upon joining the domain.

**Solution:**  $A$  generates a new master key and shares it with  $X$ .  $X$  adds  $A$  and the key to its users table.

- b. What steps are taken when a user, say  $A$ , leaves the domain.

**Solution:**  $X$  deletes its entry for the  $A$  in the users table.

- c. What steps are taken when  $X$ 's key (used to encrypt the user keys in the users table) is exposed.

**Solution:**  $X$  generates a new key, and asks all users to generate new master keys.

**16.** A domain's authentication is handled by KDC  $X$ . Tickets can have long expiry times. Consider the following:

1.  $A$  gets a post-dated ticket  $T$  from the KDC to interact with server  $B$ .
2.  $B$  changes its master key with the KDC.
3.  $A$  presents  $T$  to  $B$ .

What is the problem here? What is a solution?

**Solution:**

- **Problem:** Ticket  $T$  is encrypted with  $B$ 's old master key (shared with  $X$ ). So when  $B$  decrypts  $T$  with its current master key,  $B$  cannot make sense of the contents and will reject  $T$ .
- **One fix:** Version numbers to master keys:
  - The KDC stores for user  $A$  the current master key and its version number.
  - $B$  remembers its old master keys and their version numbers (until tickets issued under them have expired).
  - Each ticket contains the version number (unencrypted) of the master key used to encrypt the ticket. So  $B$  knows which key to use to decrypt the ticket.

### 17. Authentication protocols

This problem has independent parts. Each part describes an authentication protocol that Alice ( $A$ ) initiates to send a message  $m$  to Bob ( $B$ ), and then asks one or more questions.

- The first question lists some properties: **confidentiality, integrity, authenticity, non-repudiation, none** and **broken**. Circle all of the *first four* properties that hold for the message. Circle **none** if *none* of the first four properties hold. Circle **broken** if the protocol requires Alice or Bob to do something they cannot (e.g., decrypt a message without the key); in this case, ignore the other properties and any additional questions in that problem.
- The additional questions, if any, have **true/false** answers.

Unless otherwise stated, symmetric keys are strong, and the attacker can eavesdrop and tamper with messages.

The following conventions are as in the slides:

$[sk_A, pk_A]$	Alice's public-key pair. Bob has $pk_A$ .
$[sk_B, pk_B]$	Bob's public-key pair. Alice has $pk_B$ .
$E_p(pk, x)$	public-key encryption of $x$ with public key $pk$
$Sgn(sk, x)$	public-key signing of $x$ with secret key $sk$
$E(s, x)$	symmetric-key encryption of $x$ in CBC mode using AES with key $s$
$D(s, x)$	symmetric-key decryption of $x$ in CBC mode using AES with key $s$
$MAC(s, x)$	symmetric-key MAC (ECBC) of $x$ using key $s$
$H(x)$	SHA-256 hash function of $x$
$HMAC(k, x)$	HMAC of $x$ using key $k$ and $H$

## 17.1.

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A: generate a new symmetric key  $s$   
 send  $[E_P(pk_B, s), E(s, m),$   
 $Sgn(sk_A, H(m))]$

B: receive message  
 extract  $m$

---

- a. Circle all that hold: **confidentiality** **integrity** **authenticity** **non-repudiation** **none** **broken**

**Solution:**

- confidentiality: Yes. *// s is new, m is encrypted by s, s is encrypted by  $pk_B$*
- integrity: Yes. *//  $H(m)$  is signed by  $sk_A$ , so any change to  $E(s, m)$  is detected.*
- authenticity: Yes. *//  $H(m)$  is signed by  $sk_A$*
- non-repudiation: Yes. *//  $H(m)$  is signed by  $sk_A$*

- b. If  $s$  comes from a password and  $m$  has structure, this is vulnerable to an offline dictionary attack: **True** **False**

**Solution:** True *// Attacker sees  $E(s, m)$*

- c. This has perfect forward secrecy: **True** **False**

**Solution:** False *// If attacker gets  $sk_B$ , it can decrypt  $E(s, m)$*

17.2.  $A$  and  $B$  share a symmetric key  $s$ .

---

A: generate random  $c_A$   
 $n_A \leftarrow E(s, [1, c_A])$   
 send  $[n_A]$

B: receive message  
 $[x, c_A] \leftarrow D(s, n_A)$   
 if  $(x \neq 1)$  "FAIL"  
 generate random  $c_B$   
 $n_B \leftarrow E(s, [c_B, c_A + 1])$   
 send  $[n_B]$

A: receive message  
 $[c_B, r_A] \leftarrow D(s, n_B)$   
 if  $(r_A \neq c_A + 1)$  "FAIL"  
 $r_B \leftarrow E(s, c_B + 1)$   
 session key  $x \leftarrow c_A \oplus c_B$   
 send  $[r_B, E(x, m), \text{MAC}(x, m)]$

B: receive message  
 if  $(D(s, r_B) \neq c_B + 1)$  "FAIL"  
 extract msg  $m$

---

- a. Circle all that hold: **confidentiality** **integrity** **authenticity** **non-repudiation** **none** **broken**

**Solution:**

- confidentiality: Yes. *// m is encrypted by s, s is not exposed*
- integrity: Yes. *//  $\text{MAC}(s, m)$  assures B that m was sent by A*
- authenticity: Yes. *//  $\text{MAC}(s, m)$  assures B that m was sent by A*
- non-repudiation: No. *// to a third-party (even knowing s),  $\text{MAC}(s, m)$  could have been generated by B*

- b. If  $s$  comes from a password, this is vulnerable to a dictionary attack: **True** **False**

**Solution:** True *// Attacker sees  $E(s, [1, c_A])$  and  $E(s, [c_B, c_A + 1])$*

- c. This has perfect forward secrecy: **True** **False**

**Solution:** False *// If attacker gets s, it can get x and decrypt  $E(x, m)$*

**17.3.**  $A$  and  $B$  do Diffie-Helman with parameters  $p$  and  $g$ .

---

$A$ : generate random  $x$   
 $T_A \leftarrow g^x \text{ mod } p$   
 send  $[T_A]$

$A$ : receive message  
 $s \leftarrow T_B^x$   
 send  $[E(s, m)]$

$B$ : receive message  
 generate random  $y$   
 $T_B \leftarrow g^y \text{ mod } p$   
 $s \leftarrow T_A^y$   
 send  $[T_B]$

$B$ : receive message  
 extract msg  $m$

---

Circle all that hold: **confidentiality**   **integrity**   **authenticity**   **non-repudiation**   **none**   **broken**

**Solution:** none

// man-in-the-middle attack (see solution to 12b)

**17.4.** Repeat 17.4 assuming the attacker can only eavesdrop (but not tamper).

Circle all that hold: **confidentiality**   **integrity**   **authenticity**   **non-repudiation**   **none**   **broken**

**Solution:**

- confidentiality: Yes.
- integrity: Yes.
- authenticity: Yes.
- non-repudiation: No.

// see solution to 12a

// Attacker can only eavesdrop

// Attacker can only eavesdrop

// to a third-party,  $B$  could have generated msg  $m$