Classification with Nearest Neighbors

CMSC 422

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Law of Large Numbers

Suppose that $v_1, v_2, \ldots, v_N$ are independent and identically distributed random variables.

The empirical sample average approaches the population average as the number of sample goes to infinity.

$$\Pr \left( \lim_{N \to \infty} \frac{1}{N} \sum_n v_n = \mathbb{E}[v] \right) = 1$$
Today’s Topics

• Nearest Neighbors (NN) algorithms for classification
  – K-NN, Epsilon ball NN

• Fundamental Machine Learning Concepts
  – Decision boundary
Intuition for Nearest Neighbor Classification

This “rule of nearest neighbor” has considerable elementary intuitive appeal and probably corresponds to practice in many situations. For example, it is possible that much medical diagnosis is influenced by the doctor’s recollection of the subsequent history of an earlier patient whose symptoms resemble in some way those of the current patient.

(Fix and Hodges, 1952)
Intuition for Nearest Neighbor Classification

• Simple idea
  – Store all training examples
  – Classify new examples based on most similar training examples
K Nearest Neighbor Classification

**Training Data**

K: number of neighbors that classification is based on

Test instance with unknown class in \{−1; +1\}

**Algorithm 3 KNN-Predict(D, K, \(\hat{x}\))**

1: \(S \leftarrow [\ ]\)
2: for \(n = 1\) to \(N\) do
3: \(S \leftarrow S \oplus \langle d(x_n, \hat{x}), n \rangle\) \hspace{1cm} // store distance to training example \(n\)
4: end for
5: \(S \leftarrow \text{sort}(S)\) \hspace{1cm} // put lowest-distance objects first
6: \(\hat{y} \leftarrow 0\)
7: for \(k = 1\) to \(K\) do
8: \(\langle \text{dist}, n \rangle \leftarrow S_k\) \hspace{1cm} // \(n\) this is the \(k\)th closest data point
9: \(\hat{y} \leftarrow \hat{y} + y_n\) \hspace{1cm} // vote according to the label for the \(n\)th training point
10: end for
11: return \(\text{SIGN}(\hat{y})\) \hspace{1cm} // return +1 if \(\hat{y} > 0\) and −1 if \(\hat{y} < 0\)
2 approaches to learning

**Eager learning**
(eg decision trees)

- Learn/Train
  - Induce an **abstract model** from data
- Test/Predict/Classify
  - Apply learned model to new data

**Lazy learning**
(eg nearest neighbors)

- Learn
  - **Just store data** in memory
- Test/Predict/Classify
  - Compare new data to stored data

**Properties**
- Retains all information seen in training
- Complex hypothesis space
- Classification can be very slow
Components of a k-NN Classifier

• Distance metric
  – How do we measure distance between instances?
  – Determines the layout of the example space

• The k hyperparameter
  – How large a neighborhood should we consider?
  – Determines the complexity of the hypothesis space
Distance metrics

• We can use any distance function to select nearest neighbors.
• Different distances yield different neighborhoods

L2 distance
( = Euclidean distance)

L1 distance

Max norm
Decision Boundary of a Classifier

• is the line that separates positive and negative regions in the feature space

• Why is it useful?
  – it helps us visualize how examples will be classified for the entire feature space
  – it helps us visualize the complexity of the learned model
Decision Boundaries for 1-NN
Decision Boundaries change with the distance function
Decision Boundaries change with K
The k hyperparameter

• Tunes the complexity of the hypothesis space
  – If k = 1, every training example has its own neighborhood
  – If k = N, the entire feature space is one neighborhood!
• Higher k yields smoother decision boundaries
• How would you set k in practice?
What is the inductive bias of k-NN?

• Nearby instances should have the same label

• All features are equally important

• Complexity is tuned by the k parameter
Variations on k-NN: Weighted voting

• Weighted voting
  – Default: all neighbors have equal weight
  – Extension: weight neighbors by (inverse) distance
Variations on k-NN: Epsilon Ball Nearest Neighbors

• Same general principle as K-NN, but change the method for selecting which training examples vote

• Instead of using K nearest neighbors, use all examples $x$ such that

$$\text{distance}(\hat{x}, x) \leq \varepsilon$$
Exercise: How would you modify KNN-Predict to perform Epsilon Ball NN?

Algorithm 3 KNN-Predict(D, K, \( \hat{x} \))

1. \( S \leftarrow [ ] \)
2. for \( n = 1 \) to \( N \) do
3. \( S \leftarrow S \oplus \langle d(x_n, \hat{x}), n \rangle \) // store distance to training example \( n \)
4. end for
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7. for \( k = 1 \) to \( K \) do
8. \( \langle \text{dist}, n \rangle \leftarrow S_k \) // \( n \) this is the \( k \)th closest data point
9. \( \hat{y} \leftarrow \hat{y} + y_n \) // vote according to the label for the \( n \)th training point
10. end for
11. return \( \text{sign}(\hat{y}) \) // return +1 if \( \hat{y} > 0 \) and −1 if \( \hat{y} < 0 \)
Exercise: When are DT vs kNN appropriate?

<table>
<thead>
<tr>
<th>Properties of classification problem</th>
<th>Can Decision Trees handle them?</th>
<th>Can K-NN handle them?</th>
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<td>Robust to noisy training examples</td>
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<td>Fast classification is crucial</td>
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<td>Many irrelevant features</td>
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<td>Relevant features have very different scale</td>
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