Neural Networks

CMSC 422

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XOR slides by Graham Neubig (CMU)

Neural Networks

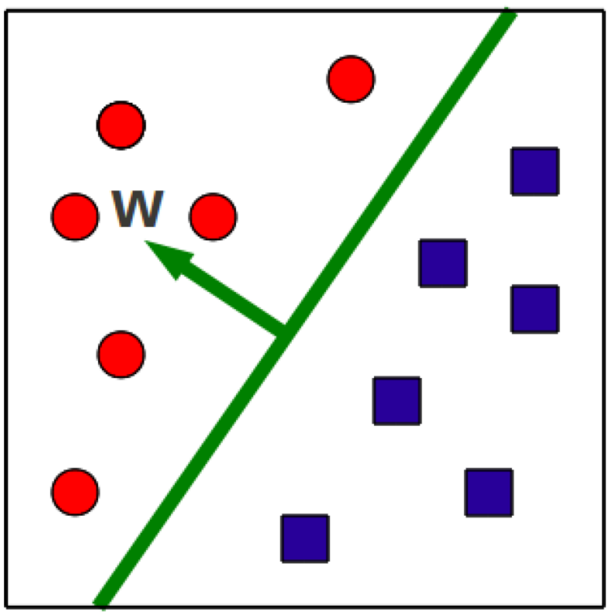
- Today
  - What are Neural Networks?
  - How to make a prediction given an input?
  - Why are neural networks powerful?
- Thursday
  - how to train them?

A warm-up example

sentiment analysis for movie review

- the movie was horrible +1
- the actors are excellent -1
- the movie was not horrible -1
- he is usually an excellent actor, but not in this movie +1

Binary classification via hyperplanes



- At test time, we check on what side of the hyperplane examples fall

$$\hat{y} = sign(w^T x + b)$$

Function Approximation with Perceptron

Problem setting

- Set of possible instances X
  - Each instance $x \in X$ is a feature vector $x = [x_1, \dots, x_D]$
- Unknown target function $f: X \rightarrow Y$
  - Y is binary valued {-1; +1}
- Set of function hypotheses $H = \{h \mid h: X \rightarrow Y\}$
  - Each hypothesis h is a hyperplane in D-dimensional space

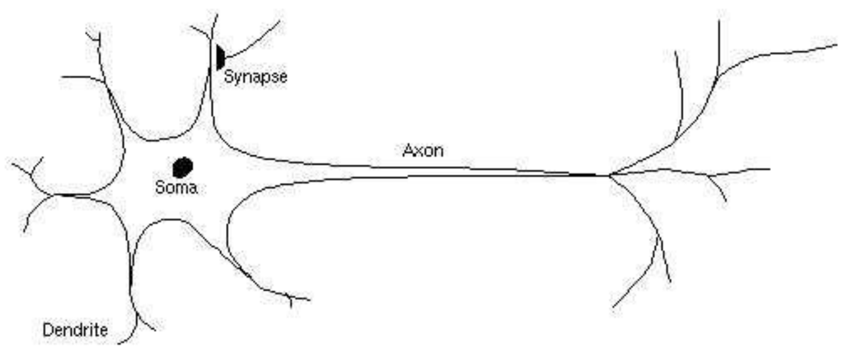
Input

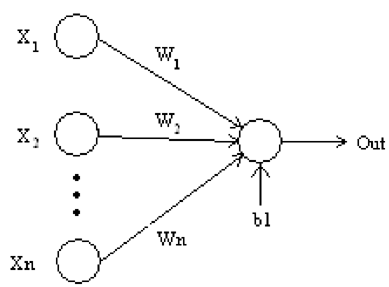
- Training examples $\{(x^{(1)}, y^{(1)}), \dots (x^{(N)}, y^{(N)})\}$ of unknown target function f

Output

- Hypothesis $h \in H$ that best approximates target function f

Aside: biological inspiration





Analogy: the perceptron as a neuron

Neural Networks

- We can think of neural networks as combination of multiple perceptrons
  - Multilayer perceptron
- Why would we want to do that?
  - Discover more complex decision boundaries
  - Learn combinations of features

What does a 2-layer perceptron look like?

(illustration on board)

- Key concepts:
  - Input dimensionality
  - Hidden units
  - Hidden layer
  - Output layer
  - Activation functions

Activation functions

- Activation functions are **non-linear** functions
  - sign function as in the perceptron
  - hyperbolic tangent and other sigmoid functions that approximate sign but are differentiable

- What happens if the hidden units use the identify function as an activation function?

Matrix of hidden layer parameters

Vector of output layer parameters

Algorithm 24 TwoLayerNetworkPredict($\mathbf{W}$, $\boldsymbol{v}$, $\hat{\boldsymbol{x}}$)

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for $i = 1$ **to** *number of hidden units* **do**
  $h_i \leftarrow \tanh(\boldsymbol{w}_i \cdot \hat{\boldsymbol{x}})$ // compute activation of hidden unit i
end for
return $\boldsymbol{v} \cdot \boldsymbol{h}$ // compute output unit

What functions can we approximate with a 2 layer perceptron?

Problem setting

- Set of possible instances X
  - Each instance $x \in X$ is a feature vector $x = [x_1, \dots, x_D]$
- Unknown target function $f: X \rightarrow Y$
  - Y is binary valued {-1; +1}
- **Set of function hypotheses** $H = \{h \mid h: X \rightarrow Y\}$

Input

- Training examples $\{(x^{(1)}, y^{(1)}), \dots (x^{(N)}, y^{(N)})\}$ of unknown target function f

Output

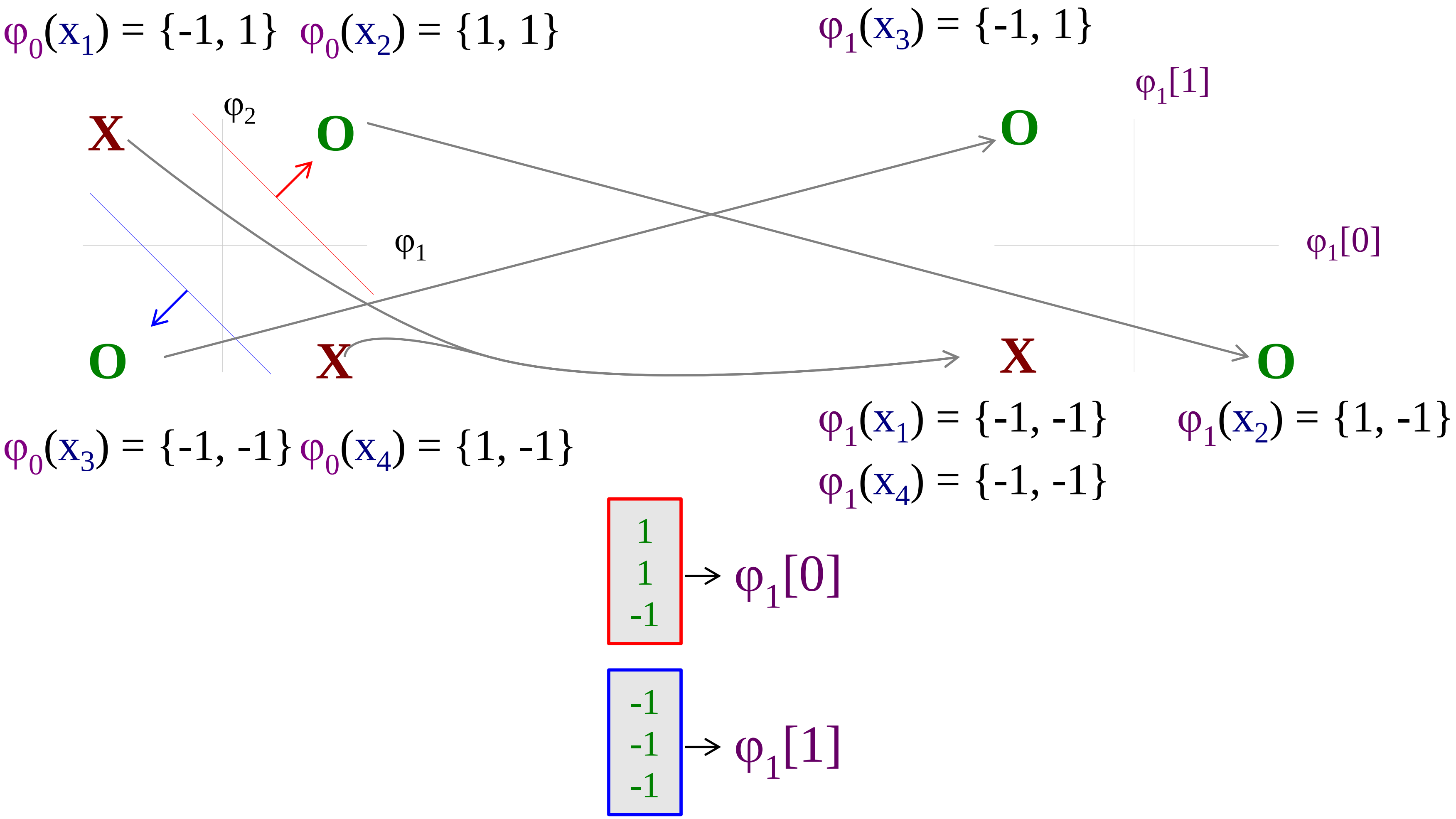
- Hypothesis $h \in H$ that best approximates target function f

Two-Layer Networks are Universal Function Approximators

- Theorem (Th 9 in CIML):

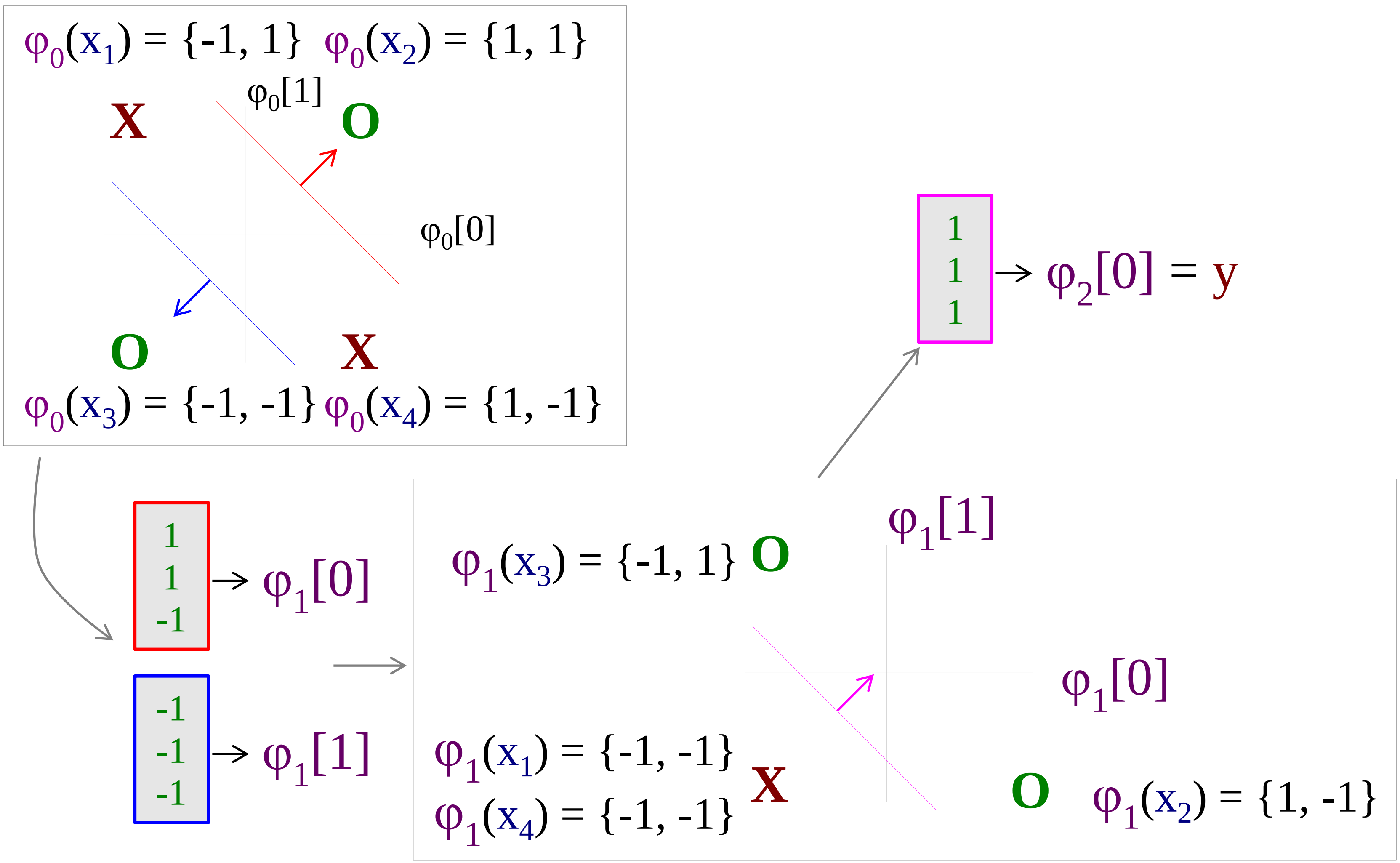
Let F be a continuous function on a bounded subset of D-dimensional space. Then there exists a two-layer neural network $\hat{F}$ with a finite number of hidden units that approximates F arbitrarily well. Namely, for all x in the domain of F, $\left|F(\boldsymbol{x}) - \hat{F}(\boldsymbol{x})\right| < \epsilon$

Example: a neural network to solve the XOR problem



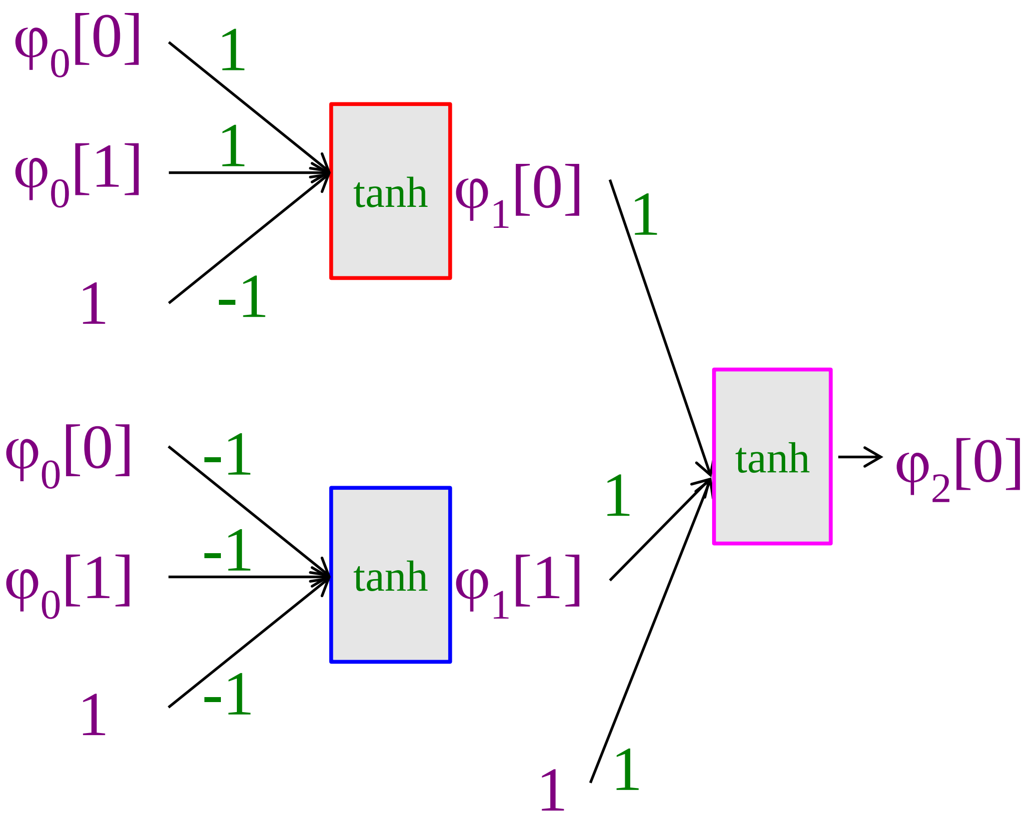
Example

- In new space, the examples are linearly separable!



Example

- The final net



Discussion

- 2-layer perceptron lets us
  - Discover more complex decision boundaries than perceptron
  - Learn combinations of features that are useful for classification

- Key design question
  - How many hidden units?
  - More hidden units yield more complex functions
  - Fewer hidden units requires fewer examples to train

Neural Networks

- Today
  - What are Neural Networks?
    - Multilayer perceptron
  - How to make a prediction given an input?
    - Simple matrix operations + non-linearities
  - Why are neural networks powerful?
    - Universal function approximators!

- Next
  - How to train them?