Instructions

This exam contains 7 pages, including this one. Make sure you have all the pages. Write your name on the top of this page before starting the exam.

Write your answers on the exam sheets. If you finish at least 15 minutes early, bring your exam to the front when you are finished; otherwise, wait until the end of the exam to turn it in. Please be as quiet as possible.

If you have a question, raise your hand. If you feel an exam question assumes something that is not written, write it down on your exam sheet. Barring some unforeseen error on the exam, however, you shouldn’t need to do this at all, so be careful when making assumptions.
Question 1. Short Answer (25 points).

a. (5 points) Briefly explain the difference between a compiler and an interpreter.

Answer: An interpreter is a program that executes another program. A compiler translates a program from a source language to a target language, and the target language program can then be executed either by an interpreter or by an actual CPU.

b. (5 points) In the context of lexing and parsing, briefly explain what a token is and how it differs from a character in the language to be parsed.

Answer: Tokens are output by the lexer, and may be either single characters in the source language or sequences of characters that form logical "words" that will be consumed by the parser. Tokens can also represent sets of words, e.g., all integers may lex to the same token (of course, the parser can retrieve the underlying distinct values as needed). The parser itself consumes a stream of input tokens, rather than source language characters.
c. **(5 points)** Briefly explain what makes a language *call-by-value*. Is OCaml a call-by-value language?

Answer: *Call-by-value means function arguments must be fully evaluated before a function is called. OCaml is indeed a call-by-value language.*

d. **(10 points)** Recall the type of tries from project 1:

```ocaml
type ('k, 'v) trie = Trie of 'v option * (('k * ('k, 'v) trie) list)
```

Write a function `find t ks` that returns the value `k` is mapped to in `t`, or aborts with `Not_found` if `ks` is not mapped in `t`. Feel free to use any functions from the `List` module you like.

Answer: *let rec find t ks = match t, ks with
  | Trie(Some v, _), [] -> v
  | Trie(None, _), [] -> raise Not_found
  | Trie(_, ts), k::ks -> find (List.assoc k ts) ks*
Question 2. Parsing (40 points).

a. (10 points) Consider the following grammar and associated parsing table.

<table>
<thead>
<tr>
<th>State</th>
<th>Action</th>
<th>Goto</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>s5</td>
<td>1 2</td>
</tr>
<tr>
<td>1</td>
<td>acc</td>
<td></td>
</tr>
</tbody>
</table>

0. \( S' \rightarrow S \)
1. \( S \rightarrow aSb \)
2. \( S \rightarrow D \)
3. \( D \rightarrow dD \)
4. \( D \rightarrow d \)

Fill in the following to show how the string \( aadbb \$ \) is parsed. You may or may not need to use all the rows. Add extra rows if necessary.

<table>
<thead>
<tr>
<th>Stack</th>
<th>Input</th>
<th>Action</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td>aadbb$</td>
</tr>
</tbody>
</table>
b. (20 points) Draw the LR(1) parsing DFA for the following grammar:

\[
\begin{align*}
S & \rightarrow aBDe \\
B & \rightarrow Bb \mid b \\
D & \rightarrow d \mid \varepsilon
\end{align*}
\]

\[
\begin{align*}
S' & \rightarrow .S, $ \\
S & \rightarrow .aBDe, $ \\
S' & \rightarrow S., $
\end{align*}
\]

\[
\begin{align*}
S & \rightarrow aB.De, $\\
B & \rightarrow B.b, b/d/e \\
B & \rightarrow .b, b/d/e \\
B & \rightarrow b., b/d/e \\
D & \rightarrow .d, e \\
D & \rightarrow ., e \\
B & \rightarrow Bb., b/d/e \\
S & \rightarrow aBD.e, $ \\
S & \rightarrow aBDe., $ \\
D & \rightarrow d., e
\end{align*}
\]

c. (10 points) Draw an LR(1) DFA state, along with any needed outgoing edges, that has a shift/reduce conflict. State on which character a conflict occurs. Similarly, draw an LR(1) DFA state, along with any needed outgoing edges, that has a reduce/reduce conflict, and state on which character a conflict occurs. Be sure to label which state has the shift/reduce conflict and which state has the reduce/reduce conflict.

As an illustration, here is a state without any conflicts:

\[
\begin{align*}
S & \rightarrow .aB, $ \\
S & \rightarrow C., b
\end{align*}
\]

\[
\begin{align*}
S & \rightarrow .aB, $ \\
S & \rightarrow C., a
\end{align*}
\]

\[
\begin{align*}
S & \rightarrow a., a \\
B & \rightarrow a., a
\end{align*}
\]
Question 3. Operational Semantics (35 points.)

a. (10 points) Here are partial big-step operational semantics for arithmetic expressions

\[ a ::= n \mid X \mid a + a \]

where \( X \in \text{Var} \) ranges over variables, and a program state \( \sigma : \text{Var} \to n \) maps variables to integers \( n \).

\[
\begin{array}{llll}
\text{Int} & \langle n, \sigma \rangle \rightarrow n & \text{Var} & \langle X, \sigma \rangle \rightarrow \sigma(X) \\
\text{PLUS} & \langle a_1, \sigma \rangle \rightarrow n & \langle a_2, \sigma \rangle \rightarrow m & \langle a_1 + a_2, \sigma \rangle \rightarrow p \rightarrow n + m
\end{array}
\]

Draw a derivation showing that \( \langle 1 + (X + Y), \sigma \rangle \rightarrow 6 \) if \( \sigma = [X \mapsto 2, Y \mapsto 3] \).

b. (8 points) Here are small-step semantics rules for the same language:

\[
\begin{array}{llll}
\text{VAR} & X \rightarrow_{\sigma} \sigma(X) & \text{RIGHT} & a_2 \rightarrow_{\sigma} a'_2 \rightarrow_{\sigma} a_1 + a'_2 \rightarrow_{\sigma} a_1 + n \rightarrow_{\sigma} a'_1 + n \rightarrow_{\sigma} p = n + m & \text{PLUS} & p = n + m \rightarrow_{\sigma} p
\end{array}
\]

Show that \( 1 + (X + Y) \rightarrow_{\sigma} 6 \) by showing each step of the reduction, where \( \sigma = [X \mapsto 2, Y \mapsto 3] \). You don’t need to show the derivations that lead to the individual steps, just the steps themselves.

c. (2 points) Does the above small-step semantics evaluate the left- or right-hand side of a summation first? Briefly justify your answer.
d. (10 points) Here I’ve tried to write down the big-step operational semantics for commands:

\[ c ::= \text{skip} \mid X := a \mid c; c \mid \text{if } b \text{ then } c \text{ else } c \mid \text{while } b \text{ do } c \]

But I did a terrible job. List all the mistakes I made, assuming I meant to write a complete, standard semantics for this language. (You don’t need to rewrite the semantics, just enumerate my mistakes.)

<table>
<thead>
<tr>
<th>Rule</th>
<th>Original Rule</th>
<th>Corrected Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Assign</strong></td>
<td>( \langle a, \sigma \rangle \rightarrow n ) \langle X := a, \sigma \rangle \rightarrow \sigma \left[ X \mapsto n \right] )</td>
<td>( \langle a, \sigma \rangle \rightarrow n ) \langle X := a, \sigma \rangle \rightarrow \sigma \left[ X \mapsto n \right] )</td>
</tr>
<tr>
<td><strong>Seq</strong></td>
<td>( \langle c_0, \sigma_1 \rangle \rightarrow \sigma_0 ) \langle c_1, \sigma \rangle \rightarrow \sigma_1 )</td>
<td>( \langle c_0, \sigma_1 \rangle \rightarrow \sigma_0 ) \langle c_1, \sigma \rangle \rightarrow \sigma_1 )</td>
</tr>
<tr>
<td><strong>If-T</strong></td>
<td>( \langle b, \sigma \rangle \rightarrow \text{true} ) \langle c_0, \sigma \rangle \rightarrow \sigma' )</td>
<td>( \langle b, \sigma \rangle \rightarrow \text{true} ) \langle c_0, \sigma \rangle \rightarrow \sigma' )</td>
</tr>
<tr>
<td><strong>If-F</strong></td>
<td>( \langle b, \sigma \rangle \rightarrow \text{false} ) \langle c_0, \sigma \rangle \rightarrow \sigma' )</td>
<td>( \langle b, \sigma \rangle \rightarrow \text{false} ) \langle c_0, \sigma \rangle \rightarrow \sigma' )</td>
</tr>
<tr>
<td><strong>While-T</strong></td>
<td>( \langle b, \sigma \rangle \rightarrow \text{true} ) \langle \text{while } b \text{ do } c, \sigma \rangle \rightarrow \sigma )</td>
<td>( \langle b, \sigma \rangle \rightarrow \text{true} ) \langle \text{while } b \text{ do } c, \sigma \rangle \rightarrow \sigma )</td>
</tr>
<tr>
<td><strong>While-F</strong></td>
<td>( \langle b, \sigma \rangle \rightarrow \text{false} ) \langle c, \sigma \rangle \rightarrow \sigma' )</td>
<td>( \langle b, \sigma \rangle \rightarrow \text{false} ) \langle c, \sigma \rangle \rightarrow \sigma' )</td>
</tr>
</tbody>
</table>

e. (5 points) Suppose we extend our language with a new form “repeat until b” that executes c until b becomes true, at which point the loop exits. (Note this means c will always be executed at least once.) Write down big-step operational rule(s) for this new form. (Assume there exist other big-step rules of the form \( \langle c, \sigma \rangle \rightarrow \sigma' \) for commands, and \( \langle b, \sigma \rangle \rightarrow bv \) for booleans, where \( bv \) ranges over true and false.)