Instructions

This exam contains 9 pages, including this one. Make sure you have all the pages. Write your name on the top of this page before starting the exam.

Write your answers on the exam sheets. If you finish at least 15 minutes early, bring your exam to the front when you are finished; otherwise, wait until the end of the exam to turn it in. Please be as quiet as possible.

If you have a question, raise your hand. If you feel an exam question assumes something that is not written, write it down on your exam sheet. Barring some unforeseen error on the exam, however, you shouldn’t need to do this at all, so be careful when making assumptions.

<table>
<thead>
<tr>
<th>Question</th>
<th>Score</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>23</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>42</td>
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<tr>
<td>3</td>
<td></td>
<td>35</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>100</td>
</tr>
</tbody>
</table>
Question 1. Short Answer (23 points).

a. (5 points) Briefly explain what the front-end of a compiler is.

**Answer:** The front-end is the lexer and parser, i.e., the part of the compiler that converts a string of characters into a structured representation such as an abstract syntax tree.

b. (5 points) How are OCaml **ref**s and OCaml records related?

**Answer:** A **ref** in OCaml is actually implemented as a record with a single mutable field, **contents**.
c. (5 points) Briefly explain how LALR(1) parsing and LR(1) parsing are related.

Answer: LALR(1) parsing is the same as LR(1) parsing, except the DFA is compressed. If the LR(1) parser has two states that share the same core (i.e., the LR(1) items are the same except for the lookahead), then those states are merged in the LALR(1) parser.

d. (8 points) Recall the following data types from Project 1:

```
type expr =
  EFalse
| ETrue
| EVar of string
| EAnd of expr * expr
| EOr of expr * expr
| ENot of expr
| EForall of string * expr
| EExists of string * expr
```

```
type bvec = expr list (* low order bit at head of list *)
```

Write a function `neq : bvec -> bvec -> expr` that returns an expression representing whether two vectors are not equal. You may not call `eq`. You can assume the two vectors are the same length.

Answer:

```
let rec neq v1 v2 = match v1, v2 with
  | [], [] -> EFalse
  | h1::t1, h2::t2 -> EOr(EOr(EAnd(h1, ENot h2), EAnd(ENot h1, h2)), neq t1 t2)
```
Consider the following grammar and associated parsing table.

<table>
<thead>
<tr>
<th>State</th>
<th>Action</th>
<th>Goto</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>s2</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>s3</td>
<td>acc</td>
</tr>
<tr>
<td>2</td>
<td>r2</td>
<td>r2</td>
</tr>
<tr>
<td>3</td>
<td>s4</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>r3</td>
<td>r3</td>
</tr>
<tr>
<td>5</td>
<td>r4</td>
<td>r4</td>
</tr>
<tr>
<td>6</td>
<td>r3</td>
<td>r3</td>
</tr>
<tr>
<td>7</td>
<td>r1</td>
<td>r1</td>
</tr>
</tbody>
</table>

Fill in the following to show how the string $cabbab$ is parsed. You may or may not need to use all the rows. Add extra rows if necessary.

<table>
<thead>
<tr>
<th>Stack</th>
<th>Input</th>
<th>Action</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$cabbab$</td>
<td>s2</td>
</tr>
<tr>
<td>0, c, 2</td>
<td>$ababb$</td>
<td>r2</td>
</tr>
<tr>
<td>0, S, 1</td>
<td>$ababb$</td>
<td>s3</td>
</tr>
<tr>
<td>0, S, 1, a, 3</td>
<td>$bbabb$</td>
<td>s4</td>
</tr>
<tr>
<td>0, S, 1, a, 3, b, 4</td>
<td>$bab$</td>
<td>s6</td>
</tr>
<tr>
<td>0, S, 1, a, 3, b, 4, b, 6</td>
<td>$ab$</td>
<td>r3</td>
</tr>
<tr>
<td>0, S, 1, a, 3, b, 4, B, 5</td>
<td>$ab$</td>
<td>r4</td>
</tr>
<tr>
<td>0, S, 1, a, 3, B, 7</td>
<td>$ab$</td>
<td>r1</td>
</tr>
<tr>
<td>0, S, 1</td>
<td>$ab$</td>
<td>s3</td>
</tr>
<tr>
<td>0, S, 1, a, 3</td>
<td>$b$</td>
<td>s4</td>
</tr>
<tr>
<td>0, S, 1, a, 3, b, 4</td>
<td>$$</td>
<td>r3</td>
</tr>
<tr>
<td>0, S, 1, a, 3, B, 7</td>
<td>$$</td>
<td>r1</td>
</tr>
<tr>
<td>0, S, 1</td>
<td>$$</td>
<td>acc</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$$</td>
</tr>
</tbody>
</table>
b. (25 points) Draw the LR(1) parsing DFA for the following grammar:

\[
\begin{align*}
    S & \rightarrow aBCa \mid B \\
    B & \rightarrow b \\
    C & \rightarrow \varepsilon \mid Cc
\end{align*}
\]

Answer:

![Diagram](image)
c. (5 points) I had some trouble solving Project 2. At one point, running `ocamlyacc -v` showed the following (partial) output:

```
1 main : EOF
2 | bexpr EOF
3 bexpr : ID
4 | bexpr AND bexpr

state 1
%entry% : '\001' main (5)
  EOF shift 3
  ID shift 4
  . error
  main goto 5
  bexpr goto 6

state 3
  main : EOF . (1)
  . reduce 1

state 4
  bexpr : ID . (3)
  . reduce 3

state 6
  main : bexpr . EOF (2)
  bexpr : bexpr . AND bexpr (4)
  EOF shift 7
  AND shift 8
  . error
  main goto 5
  bexpr goto 6

state 8
  bexpr : bexpr AND . bexpr (4)
  ID shift 4
  . error
  bexpr goto 9

state 9
  shift/reduce conflict
  (shift 8, reduce 4) on AND

state 7
  main : bexpr EOF . (2)
  . reduce 2

state 10
  bexpr : bexpr AND . bexpr (4)
  AND shift 8
  EOF reduce 4
```

I need help understanding what’s wrong. Write down an input that will cause the parsing DFA to reach state 9 and have to choose between the shift and the reduce that are in conflict. Then, briefly explain what problem with the grammar causes the conflict in that state.

**Answer:** `ID AND ID AND ID EOF` will reach state 9 and exhibit the conflict (note that `ID AND ID EOF` reaches state 9, but the lookahead will be `EOF` and hence it will not trigger the conflict). The problem with the grammar is that it’s ambiguous: it allows conjunctions to be associate either to the left or right.
**Question 3. Operational Semantics (35 points).**

a. (10 points) Here are partial big-step operational semantics for boolean expressions.

\[
\begin{align*}
  b & ::= \ b v \ | \ X \ | \ \neg b \ | \ b \land b \ | \ b \lor b \\
  b v & ::= \ true \ | \ false
\end{align*}
\]

where \( X \in \text{Var} \) ranges over boolean values true and false, and a program state \( \sigma : \text{Var} \to b v \) maps variables to boolean values.

\[
\begin{array}{ccc}
  \text{TRUE} & \text{FALSE} & \text{VAR} \\
  \langle \text{true}, \sigma \rangle \to \text{true} & \langle \text{false}, \sigma \rangle \to \text{false} & \langle X, \sigma \rangle \to \sigma(X) \\
  \text{AND} & \langle b_1, \sigma \rangle \to bv_1 & \langle b_2, \sigma \rangle \to bv_2 \\
  & bv = bv_1 \land bv_2 & \\
  & \langle b_1 \land b_2, \sigma \rangle \to bv & \\
  \text{OR} & \langle b_1, \sigma \rangle \to bv_1 & \langle b_2, \sigma \rangle \to bv_2 \\
  & bv = bv_1 \lor bv_2 & \\
  & \langle b_1 \lor b_2, \sigma \rangle \to bv & \\
\end{array}
\]

Draw a derivation showing that \( \langle X \land (Y \lor \text{false}), \sigma \rangle \to \text{true} \) if \( \sigma = [X \mapsto \text{true}, Y \mapsto \text{true}] \). **Label each step of the derivation with the operational semantics rule used.**

**Answer:**

\[
\begin{array}{ccc}
  \text{VAR} & \langle X, \sigma \rangle \to \text{true} & \langle Y, \sigma \rangle \to \text{true} \\
  \langle Y \lor \text{false}, \sigma \rangle \to \text{false} & \langle X \land (Y \lor \text{false}), \sigma \rangle \to \text{true} \\
\end{array}
\]

b. (5 points) One big-step rule is missing. Write it down.

**Answer:**

\[
\begin{array}{c}
  \langle Y, \sigma \rangle \to bv \\
  bv' = \neg bv \\
  \langle \neg b, \sigma \rangle \to bv'
\end{array}
\]
c. (8 points) Here are partial small-step semantics rules for the same language:

<table>
<thead>
<tr>
<th>Rule</th>
<th>Steps</th>
</tr>
</thead>
<tbody>
<tr>
<td>VAR</td>
<td>( \sigma(X) = bv )</td>
</tr>
<tr>
<td>L-AND</td>
<td>( b_1 \rightarrow \sigma b'_1 )</td>
</tr>
<tr>
<td>T-AND</td>
<td>( \text{true} \land b \rightarrow_\sigma b )</td>
</tr>
<tr>
<td>F-AND</td>
<td>( \text{false} \land b \rightarrow_\sigma \text{false} )</td>
</tr>
<tr>
<td>L-OR</td>
<td>( b_1 \rightarrow_\sigma b'_1 )</td>
</tr>
<tr>
<td>T-OR</td>
<td>( \text{true} \lor b \rightarrow_\sigma \text{true} )</td>
</tr>
<tr>
<td>F-OR</td>
<td>( \text{false} \lor b \rightarrow_\sigma b )</td>
</tr>
</tbody>
</table>

Show that \( X \land (Y \lor \text{false}) \) is true if \( \sigma = [X \leftrightarrow \text{true}, Y \leftrightarrow \text{true}] \). Show each step of the reduction, but you don’t need to show the derivations that lead to the individual steps, just the steps themselves. (The relation \( b \rightarrow_\sigma b' \) means \( b \) reaches \( b' \) in zero or more steps of \( \rightarrow_\sigma \).)

**Answer:**

\[
\begin{align*}
X \land (Y \lor \text{false}) & \rightarrow_\sigma \text{true} \land (Y \lor \text{false}) \\
& \rightarrow_\sigma \text{true} \lor \text{false} \\
& \rightarrow_\sigma \text{true} \\
& \rightarrow_\sigma \text{true}
\end{align*}
\]

d. (7 points) Suppose we extend the grammar to include implication: \( b ::= \ldots \mid b \Rightarrow b \). Write down the corresponding small-step rules, following the pattern in part c, using the normal interpretation of logical implication.

**Answer:**

<table>
<thead>
<tr>
<th>Rule</th>
<th>Steps</th>
</tr>
</thead>
<tbody>
<tr>
<td>L-IMP</td>
<td>( b_1 \rightarrow_\sigma b'_1 )</td>
</tr>
<tr>
<td>T-IMP</td>
<td>( \text{true} \Rightarrow b \rightarrow_\sigma b )</td>
</tr>
<tr>
<td>F-IMP</td>
<td>( \text{false} \Rightarrow b \rightarrow_\sigma \text{true} )</td>
</tr>
</tbody>
</table>
e. (5 points) Briefly explain what a normal form is in small-step operational semantics. What are the normal form(s) of the semantics in part c?

Answer: A normal form is a term that cannot be reduced any further. The normal forms in the operational semantics just above are true and false.