Instructions

This exam contains 9 pages, including this one. Make sure you have all the pages. Write your name on the top of this page before starting the exam.

Write your answers on the exam sheets. If you finish at least 15 minutes early, bring your exam to the front when you are finished; otherwise, wait until the end of the exam to turn it in. Please be as quiet as possible.

If you have a question, raise your hand. If you feel an exam question assumes something that is not written, write it down on your exam sheet. Barring some unforeseen error on the exam, however, you shouldn’t need to do this at all, so be careful when making assumptions.

<table>
<thead>
<tr>
<th>Question</th>
<th>Score</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>20</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>25</td>
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<tr>
<td>3</td>
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<td>20</td>
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<td>4</td>
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<td>5</td>
<td></td>
<td>15</td>
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<td>Total</td>
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<td>100</td>
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</tbody>
</table>
Question 1. Short Answer (20 points).

a. (5 points) In class we’ve discussed three virtual machines: the Lua VM, the Java VM, and the Dalvik VM. List two differences that you can observe among the VMs, e.g., something different between Lua VM and the Java VM, etc.

   **Answer:** There are too many possible valid answers to list here. The most common correct answers were: register-based (Lua, Dalvik) vs. stack-based (Java); many bytecode files (Java) vs. one bytecode file (Dalvik); and instruction-level support for tables (Lua) vs. not (Java).

b. (5 points) Is dataflow analysis guaranteed to compute the *meet over all paths* (MOP) solution? Explain your answer.

   **Answer:** Not necessarily. For a distributive dataflow analysis problem, in which for any transfer function $f$ it is the case that $f(x \sqcap y) = f(x) \sqcap f(y)$, then data flow will compute the MOP solution because meet loses no information. But for non-distributive problems, such as constant propagation, data flow analysis may not compute the MOP solution. Additionally, data flow analysis may not terminte, in which case it does not compute any solution.
c. (5 points) What are the three possible representations of two-dimensional arrays that we discussed in class? Explain the differences between them. Feel free to draw pictures if it’s helpful.

Answer: In row-major order, the array is represented as a single, contiguous block of memory, with all elements of the first row at the beginning, then all elements of the second row, etc. Column-major order is similar, except all elements of the first column come first, then all elements of the second column etc. With indirection vectors, the array is actually an array of pointers, one per row, pointing to another array somewhere else in memory containing the row. Thus, with indirection vectors the memory is not necessarily contiguous. (Indirection vectors could be column-oriented as well.)

d. (5 points) What is a virtual method table?

Answer: In a class-based language, each instance of a class shares the same methods. So rather than replicate that list of classes for each instance, the compiler creates a single virtual method table (vtable) per class containing its methods. Then instances have use a pointer to their class vtable to perform dynamic dispatch.
Question 2. Types (25 points).

a. (7 points) Suppose `int` is a subtype of `float`, and consider the following four types:

\[
\begin{align*}
\text{int} \to \text{int} & \quad \text{int} \to \text{float} & \quad \text{float} \to \text{int} & \quad \text{float} \to \text{float}
\end{align*}
\]

Draw a partial order diagram showing the \( \leq \) relationships between these four types implied by the standard subtyping rules. For example, if we asked you to draw the relationship between `int` and `float`, you would draw the following: 

\[
\begin{array}{c}
\text{int} \\
\text{float}
\end{array}
\]

Also indicate which element is \( \top \), and which element is \( \bot \), if any.

**Answer:**

\[
\begin{array}{c}
\text{int} \to \text{float} \\
\text{float} \to \text{float} \\
\text{int} \to \text{int}
\end{array}
\]

\( \top = \text{int} \to \text{float} \) and \( \bot = \text{float} \to \text{int} \).

b. (8 points) Write down every type \( t \) such that \( ((\text{int} \to \text{float}) \to \text{float}) \leq t \), following standard subtyping rules.

**Answer:**

\[
\begin{align*}
(i \to i) & \to f \\
(i \to f) & \to f \\
(f \to i) & \to f \\
(f \to f) & \to f
\end{align*}
\]
c. (10 points) Fill in the following table with either an untyped (i.e., no type parameter annotations) lambda calculus term (on the left) or its corresponding type according to the type inference algorithm we saw in class (on the right). We’ve filled in the first row as an example. Remember the scope of \( \lambda \) extends as far to the right as possible. For example, \( \lambda x.\lambda y.x \) is parsed as \( \lambda x.\lambda y.(x \ y) \).

<table>
<thead>
<tr>
<th>Term</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda x.x )</td>
<td>( \alpha \to \alpha )</td>
</tr>
<tr>
<td>( \lambda x.3 )</td>
<td>( \alpha \to \text{int} )</td>
</tr>
<tr>
<td>( \lambda x.\lambda y.x )</td>
<td>( \alpha \to \beta \to \alpha )</td>
</tr>
<tr>
<td>( \lambda x.\lambda y.x \ y )</td>
<td>( (\alpha \to \beta) \to \alpha \to \beta )</td>
</tr>
<tr>
<td>( \lambda x.\lambda y.\lambda z.x \ y \ z )</td>
<td>( (\beta \to \gamma) \to (\alpha \to \beta) \to \alpha \to \gamma )</td>
</tr>
<tr>
<td>( \lambda x.\lambda y.\lambda z.x \ y \ z )</td>
<td>( (\alpha \to \beta \to \gamma) \to (\alpha \to \beta) \to \alpha \to \gamma )</td>
</tr>
<tr>
<td>( \lambda x. x (\lambda y.3) )</td>
<td>( ((\alpha \to \text{int}) \to \beta) \to \beta )</td>
</tr>
<tr>
<td>( \lambda x.\lambda y.x \ (x \ y) )</td>
<td>( (\alpha \to \alpha) \to \alpha \to \alpha )</td>
</tr>
</tbody>
</table>
**Question 3. Data flow analysis (20 points).** Consider the following control-flow graph.

![Control-flow graph](image)

**a. (10 points)** Write down the sets of live variables at the beginning of each statement.

<table>
<thead>
<tr>
<th>Statement</th>
<th>Variables live at beginning of statement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a, b</td>
</tr>
<tr>
<td>2</td>
<td>a, b, c</td>
</tr>
<tr>
<td>3</td>
<td>a, b, d</td>
</tr>
<tr>
<td>4</td>
<td>a, b, d</td>
</tr>
<tr>
<td>5</td>
<td>a, b, d, i</td>
</tr>
<tr>
<td>6</td>
<td>a, b, d, i</td>
</tr>
<tr>
<td>7</td>
<td>a, b, c, d</td>
</tr>
<tr>
<td>8</td>
<td>a, b, c, d</td>
</tr>
<tr>
<td>9</td>
<td>a, b, d</td>
</tr>
<tr>
<td>10</td>
<td>a, b, d</td>
</tr>
</tbody>
</table>
b. **(10 points)** Write down the set of available expressions at the *end* of each statement.

<table>
<thead>
<tr>
<th>Statement</th>
<th>Expressions available at end of statement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a+b</td>
</tr>
<tr>
<td>2</td>
<td>a+b, a*c</td>
</tr>
<tr>
<td>3</td>
<td>a+b, a<em>c, d</em>d</td>
</tr>
<tr>
<td>4</td>
<td>a+b, a<em>c, d</em>d</td>
</tr>
<tr>
<td>5</td>
<td>a+b, d*d</td>
</tr>
<tr>
<td>6</td>
<td>a+b, d*d, d+i</td>
</tr>
<tr>
<td>7</td>
<td>a+b, d*d, d+i</td>
</tr>
<tr>
<td>8</td>
<td>a+b, d<em>d, d+i, a</em>c</td>
</tr>
<tr>
<td>9</td>
<td>a+b, d*d, d+i</td>
</tr>
<tr>
<td>10</td>
<td>a+b, d*d, d+1</td>
</tr>
</tbody>
</table>
Question 4. Data flow analysis design (20 points). The goal of sign analysis is to determine, for each variable in the program, whether it is positive, negative, or zero. In this problem, you will design a data flow analysis that implements sign analysis.

a. (5 points) Should the analysis be forward or backward?

   Answer: Forward.

b. (5 points) What should be the lattice for sign analysis? Draw a picture to help your explanation. Note: You do not need to worry about various combinations of signs (e.g., “positive or zero”).

   Answer: The lattice is just like constant propagation, except instead of mapping variables to values, it maps them to signs. More precisely, for each variable $a$, construct a lattice $L_a$ with as:

   \[
   L_a = \begin{array}{c}
   \top \\
   +
   \\
   0
   \\
   -
   \\
   \bot
   \end{array}
   \]

   Then the sign analysis lattice is $L_a \times L_b \times \ldots$ where $a, b, \ldots$ are the program variables.

c. (5 points) Suppose $x = +$, $y = -$, and $z = 0$ just prior to each for the following statements. Write down the dataflow fact about $a$ just after the statement.

<table>
<thead>
<tr>
<th>Statement</th>
<th>Value for $a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a := x + x$</td>
<td>$a = +$</td>
</tr>
<tr>
<td>$a := x \ast y$</td>
<td>$a = -$</td>
</tr>
<tr>
<td>$a = x - y$</td>
<td>$a = +$</td>
</tr>
<tr>
<td>$a := x + y$</td>
<td>$a = \bot$</td>
</tr>
</tbody>
</table>

d. (5 points) If we implement the usual dataflow analysis algorithm, is the algorithm guaranteed to terminate? Why or why not?

   Answer: Yes, the lattice has finite height and the transfer functions are monotonic, so the analysis will terminate.
Question 5. Code generation (15 points). Below is a snippet of `08-codegen-2.ml` from class, showing the input expression language, the “bytecode” instruction language, and compilation. To save some writing, we renamed ‘L_Register’ to ‘L_Reg.

```
type expr =
           | EInt of int
           | EAdd of expr * expr
           | ESub of expr * expr
           | EMul of expr * expr
           | Eid of string
           | EAssn of string * expr
           | ESeq of expr * expr
           | EIfZero of expr * expr * expr

let rec comp_expr (st:(string * int) list) =
    function
    | EInt n →
      let r = next_reg () in
      (r, [ILoad ('L_Reg r, 'L_Int n)])
    | EIfZero (e1, e2, e3) →
      let (r1, p1) = comp_expr st e1 in
      let (r2, p2) = comp_expr st e2 in
      let (r3, p3) = comp_expr st e3 in
      let r = next_reg () in
      (r, [ILoad ('L_Register r, 'L_Int 0)]) @
      [IlfZero ('L_Register r1, 2+(List.length p3))] @
      [IMov ('L_Register r, 'L_Register r1);
       IJmp (1+(List.length p2))]
      @
      [IMov ('L_Register r, 'L_Register r2)]
      @
      p1 @ p2 @ p3
```

Suppose we extend the source language with a for loop `EFor(e1, e2, e3, e4)` roughly corresponding to the C construct for `(e1; e2; e3) e4`. Here e1 is the initialization, e2 is the guard, e3 is the increment, and e4 is the loop body. The loop body should be executed if the guard is non-zero. The whole construct should return 0 as a result.

Write a case of `comp_expr` that compiles `EFor`.

```
let rec comp_expr (st:(string * int) list) = function

| EFor (e1, e2, e3, e4) →
```

**Answer:**

```
let (r1, p1) = comp_expr st e1 in
let (r2, p2) = comp_expr st e2 in
let (r3, p3) = comp_expr st e3 in
let (r4, p4) = comp_expr st e4 in
let p3_p4_len = (List.length p3) + (List.length p4) in
let r = next_reg () in
(r, p1 @ p2 @
 [IlfZero ('L_Register r2, p3_p4_len + 1)] @
 p4 @
 p3 @
 [IJump (-p3_p4_len + (List.length p2) + 2)] @
 [ILoad ('L_Register r, 'L_Int 0)])
```