# PUBLIC KEY CRYPTO

#### **CMSC 414** MAR 27 2018



## **RECAP: SYMMETRIC KEY CRYPTO**



#### CONFIDENTIALITY

Block ciphers

Deterministic  $\Rightarrow$  use IVs Fixed block size  $\Rightarrow$  use encryption "modes"

#### INTEGRITY

Message Authentication Codes (MACs)

Send (message, tag) pairs Verify that they match



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### **BLACKBOX #4: DIFFIE HELLMAN KEY ESTABLISHMENT**

 $x \mod N$ 

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g is a generator of mod N if{1, 2, ..., N-1} = { $g^0 \mod N, g^1 \mod N, ..., g^{N-2} \mod N$ }

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N=5, g=33<sup>0</sup> mod 5 = 1 3<sup>1</sup> mod 5 = 3 3<sup>2</sup> mod 5 = 4 3<sup>3</sup> mod 5 = 2

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Given g and g<sup>x</sup> mod N it is *infeasible* to compute x Discrete log problem





. . . . . . . . . . . .



. . . . . . . . . . .

Public knowledge: g and N



. . . . . . . .

Public knowledge: g and N



Public knowledge: g and N

Pick random a



Public knowledge: g and N

Pick random a



Public knowledge: g and N

Pick random a

 $g^a \mod N$ 

 $\sum_{g^a \mod N} g^a \mod N$ 

g N g<sup>a</sup> mod N

Public knowledge: g and N

Pick random a

ag N

 $g^a \mod N$ 

 $\sum_{g^a \mod N} g^a \mod N$ 

g N b g<sup>a</sup> mod N

#### Public knowledge: g and N

Pick random a

ag N

 $g^a \mod N$ 

Pick random b

#### **DIFFIE-HELLMAN KEY EXCHANGE** $\sum g N$ $g^a \mod N$ ag N g N b $g^a \mod N$ *Public knowledge: g and N* Pick random a $g^a \mod N$ Pick random b $g^b \mod N$





g N b g<sup>a</sup> mod N

Public knowledge: g and N







g N g<sup>a</sup> mod N g<sup>b</sup> mod N

g<sup>ab</sup> mod N

Note that just multiplying  $g^a$  and  $g^b$  won't suffice:  $g^a \mod N * g^b \mod N = g^{a+b} \mod N$ Key property: An eavesdropper cannot infer the shared secret ( $g^{ab}$ ). But what about active intermediaries?

g N g<sup>a</sup> mod N g<sup>b</sup> mod N

 $g^{ab} \mod N$ 

Given g and g<sup>x</sup> mod N it is *infeasible* to compute x Discrete log problem

Note that just multiplying  $g^a$  and  $g^b$  won't suffice:  $g^a \mod N * g^b \mod N = g^{a+b} \mod N$ 

Key property: An eavesdropper cannot infer the shared secret (g<sup>ab</sup>). But what about active intermediaries?

The attacker can interpose between the two communicating parties and insert, delete, and modify messages.



Pick random a



Pick random **x** 



Pick random b













The attacker can interpose between the two communicating parties and insert, delete, and modify messages.



The attacker can now eavesdrop on the conversation. Key property: Diffie-Hellman is *not* resilient to a MITM attack

#### TO FIX THIS PROBLEM WE NEED . . .

### **BLACKBOX #5: PUBLIC KEY CRYPTOGRAPHY**

### Shortcomings of symmetric key



Establishing a pairwise key requires a **key exchange**, which requires both parties to be *online* 

#### Issue #1: Requires pairwise key exchanges

File downloads





Email / chat

One-to-many: O(N) key exchanges

All-to-all: O(N<sup>2</sup>) key exchanges

### Shortcomings of symmetric key



Establishing a pairwise key requires a **key exchange**, which requires both parties to be *online* 

#### **Issue #2: Parties must be online**

File downloads



One-to-many: O(N) key exchanges Blue user uploads a document, then goes offline (e.g., forever)

Later, a yellow user wants to get a copy; how can it know the copy is really from the blue user?

#### 

Issue #3: How do you know to whom you're talking?

## Diffie-Hellman is resilient to *eavesdropping*, but *not tampering*



#### A protocol that solves this with trust

Trent: A trusted third party


Trent: A trusted third party



 Everybody establishes a pairwise key with Trent Good: O(N) key exchanges

Trent: A trusted third party



### 1. Everybody establishes a pairwise key with Trent **Good:** *O(N) key exchanges*

2. Trent validates each user's identity; includes in message **Good:** *Authenticated communication* 

Trent: A *trusted* third party



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1. Everybody establishes a pairwise key with Trent **Good:** *O(N) key exchanges* 

2. Trent validates each user's identity; includes in message **Good:** *Authenticated communication* 

#### Bad: All messages get sent through Trent











Do not *alter* messages
Do not *forge* messages



A public key encryption scheme comprises three algorithms

### Key generation G

- Inputs
  - Source of randomness
  - Maximum key length L
- Outputs: a key pair
  - *PK* = **public key**
  - SK = secret key

This is a *randomized* algorithm (nondeterministic output)

#### Difficult to infer SK from PK Only one person should know SK; PK should be public to all

PK and SK are intrinsically bound together: for a given PK, there is a single *corresponding* SK

Example: RSA's public keys are a pair: (exponent, modulus)

A public key encryption scheme comprises three algorithms

#### Encryption E(PK, msg)

- Inputs
  - Public key PK
  - Message msg of fixed size
- Outputs: a cipher text c same size as msg

#### This is a *randomized* algorithm (vanilla RSA is deterministic; in practice, RSA-PKCS is used instead, which adds a nonce to the message)

### PK a.k.a. "Encryption key"

Anyone who knows Alice's PK can encrypt a message to her...

A public key encryption scheme comprises three algorithms

#### Decryption D(SK, c)

- Inputs
  - Secret key SK
  - Cipher text c
- Outputs: original msg

#### This is a *deterministic* algorithm

Should always return the original message

...but only Alice can decrypt that message

A public key encryption scheme comprises three algorithms

Key generation G
$$\rightarrow PK =$$
 public key $\rightarrow SK =$  secret key

#### <u>Correctness</u>

D(SK, E(PK, m)) = m



Decryption **D(SK, c)** 

→ original msg

#### **Security**

E(PK, m) should appear random (small change to (PK,m) leads to large changes to c)

E() should approximate a one-way trapdoor function: cannot invert without access to SK

Symmetric key

Email / chat



All-to-all: O(N<sup>2</sup>) key exchanges

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Generate public/private key pair (PK,SK)

Annouce PK publicly (on website, in newspaper, ...)

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All-to-all: O(N<sup>2</sup>) key exchanges



Generate public/private key pair (PK,SK)

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Obtain PK



Send c = E(PK, msg)

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Decrypt D(SK, c) = msg

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All-to-all: O(N<sup>2</sup>) key exchanges



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Decrypt D(SK, c) = msg

O(N) keys in total

### Overcoming fixed message sizes



Like block ciphers, but there are not "modes" of public key encryption

### Overcoming fixed message sizes



#### Public key operations are *sloooow!*

### Overcoming fixed message sizes



### Public key operations are *slooooow!* Symmetric key operations are fast

Generate public/private key pair (PK,SK); publicize PK

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Obtain PK

Generate symmetric key K

Compute  $c_{msg} = e(K, msg)$ 

Compute  $c_{K} = E(PK, K)$ 

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Obtain PK

Generate symmetric key K

- Symm key Compute  $c_{msg} = e(K, msg)$
- Public key Compute  $c_{K} = E(PK, K)$

Generate public/private key pair (PK,SK); publicize PK

Obtain PK Generate *symmetric* key K

Compute  $c_{msg} = e(K, msg)$ Symm key

Compute  $c_{K} = E(PK, K)$  **Now throw away K** Public key

Generate public/private key pair (PK,SK); publicize PK

Obtain PK Generate *symmetric* key K

Compute  $c_{msg} = e(K, msg)$ Symm key

Compute c<sub>K</sub> = E(PK, K) *Now throw away K* Public key Send CK || Cmsg

Generate public/private key pair (PK,SK); publicize PK

Obtain PK Generate *symmetric* key K

Symm key Compute  $C_{msg} = e(K, msg)$ 

Compute  $c_{K} = E(PK, K)$  **Now throw away K** Public key

Send CK || Cmsg

Decrypt D(SK,  $c_K$ ) = K Decrypt d(K,  $c_{msg}$ ) = msg

Generate public/private key pair (PK,SK); publicize PK

Obtain PK Generate *symmetric* key K

Compute  $c_{msg} = e(K, msg)$ Symm key

Compute c<sub>K</sub> = E(PK, K) *Now throw away K* Public key

Send CK || Cmsg

Decrypt D(SK,  $c_K$ ) = K Decrypt d(K,  $c_{msg}$ ) = msg Public key Symm key

Obtain PK Generate *symmetric* key K Compute  $c_{msg} = e(K, msg)$ Compute  $c_{K} = E(PK, K)$ Send  $c_{K} \parallel c_{msg}$ 

#### The easy key distribution of public key

The speed and arbitrary message length of symmetric key

Protocols with public key cryptography Goal: determine from whom a message came

Symmetric key



Ideally, a user (blue) could post a message (e.g., sensitive documents or a kernel update), and then go offline

And downloaders (yellow) could subsequently infer the message's authenticity without having to have already established a pairwise key with the publisher

A digital signature scheme comprises two algorithms

Signing function Sgn(SK, m)

- Inputs
  - Secret key SK
  - Fixed-length message
- Outputs: a *signature s*

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Verification function Vfy(PK, m, s)

- Inputs
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  - Message and signature
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**Deterministic algorithm** 

Anyone with the PK can verify

A digital signature scheme comprises two algorithms

Signing Sgn(SK, m)  $\rightarrow$  a signature s

Verification Vfy(PK, m, s)
→ Yes/No if valid (m,s)

#### <u>Correctness</u> Vfy(PK, m, Sgn(SK, m)) = Yes

#### **Security**

Same as with MACs: even after a chosen plaintext attack, the attacker cannot demonstrate an existential forgery

Symmetric key

File downloads



One-to-many: O(N) key exchanges Generate public/private key pair (PK,SK)

Annouce PK publicly (on website, in newspaper, ...)

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Compute sig = Sgn(SK, msg)

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Publish msg || sig

#### Symmetric key

File downloads



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Publish msg || sig

can now go offline!

#### Symmetric key

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can now go offline!

Obtain PK, msg || sig Vfy(PK, msg, sig)



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Bob can prove that a message signed by Alice is truly from Alice (even without a *pairwise* key)

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**Non-repudiation** 

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Would require having a signature that depended on each part in the body of the letter

**Non-repudiation** 

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Integrity

Would require having a signature that depended on each part in the body of the letter

**Non-repudiation** 

Would require both of the above (unforgeable signature that depends on each part of letter)