Slides adapted from Prof Carpuat and Duraiswami

CMSC 422 Introduction to Machine Learning

Lecture 18 Neural Networks I

Furong Huang / furongh@cs.umd.edu

UNIVERSITY OF MARYLAND

Neural Networks

Today

- What are Neural Networks?
- How to make a prediction given an input?
- Why are neural networks powerful?

Next Tuesday

- how to train them?

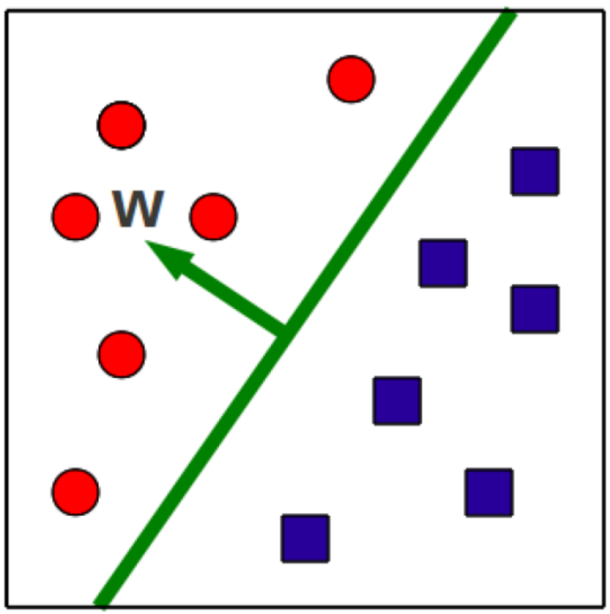
A warm-up example

sentiment analysis for movie review

the movie was horrible	+1
the actors are excellent	-1
the movie was not horrible	-1
he is usually an excellent actor, but not in this movie	+1

(on board)

Binary classification via hyperplanes



At test time, we check on what side of the hyperplane examples fall

$$\hat{y} = sign(w^T x + b)$$

Function Approximation with Perceptron

Problem setting

Set of possible instances X

Each instance $x \in X$ is a feature vector $x = [x_1, \dots, x_D]$

Unknown target function $f: X \to Y$

Y is binary valued {-1; +1}

Set of function hypotheses $H = \{h \mid h: X \to Y\}$

Each hypothesis h is a hyperplane in D-dimensional space

Input

Training examples $\{(x^{(1)}, y^{(1)}), \dots (x^{(N)}, y^{(N)})\}$ of unknown target function f

Output

Hypothesis $h \in H$ that best approximates target function f

Neural Networks

We can think of neural networks as combination of multiple perceptrons

- Multilayer perceptron

Why would we want to do that?

- Discover more complex decision boundaries
- Learn combinations of features

What does a 2-layer perceptron look like?

(illustration on board)

Key concepts:

- Input dimensionality
- Hidden units
- Hidden layer
- Output layer
- Activation functions

Activation functions

Activation functions are **non-linear** functions

- sign function as in the perceptron
- hyperbolic tangent and other sigmoid functions that approximate sign but are differentiable

What happens if the hidden units use the identify function as an activation function?

Matrix of hidden layer parameters

Vector of output layer parameters

Algorithm 24 TwoLayerNetworkPredict($\mathbf{W}, \boldsymbol{v}, \hat{\boldsymbol{x}}$)

for $i = 1$ **to** *number of hidden units* **do**
    $h_i \leftarrow \tanh(\boldsymbol{w}_i \cdot \hat{\boldsymbol{x}})$ // compute activation of hidden unit i
end for
return $\boldsymbol{v} \cdot \boldsymbol{h}$ // compute output unit

What functions can we approximate with a 2 layer perceptron?

Problem setting

Set of possible instances X

Each instance $x \in X$ is a feature vector $x = [x_1, \dots, x_D]$

Unknown target function $f: X \rightarrow Y$

Y is binary valued {-1; +1}

Set of function hypotheses $H = \{h \mid h: X \rightarrow Y\}$

Input

Training examples $\{(x^{(1)}, y^{(1)}), \dots (x^{(N)}, y^{(N)})\}$ of unknown target function f

Output

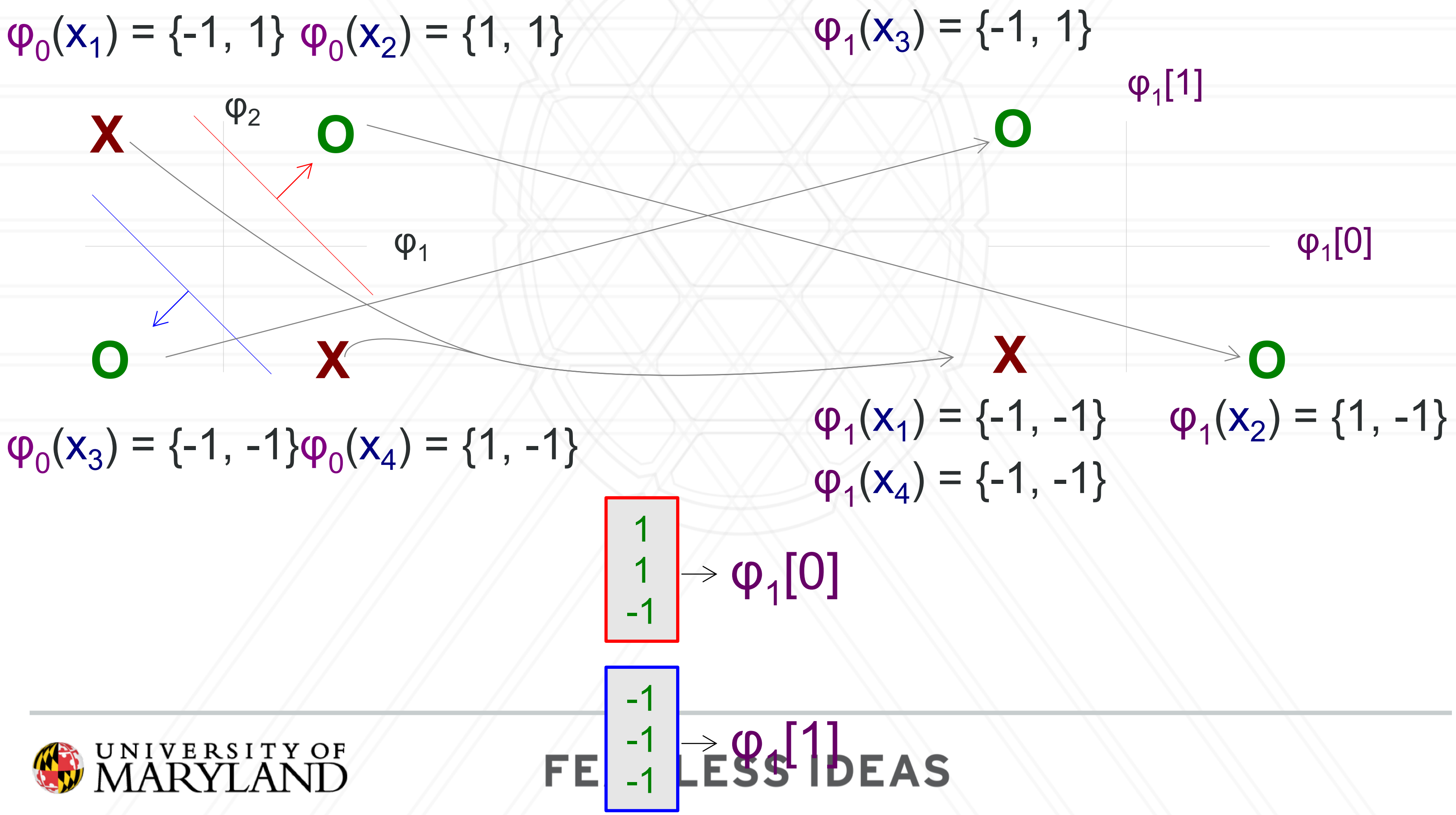
Hypothesis $h \in H$ that best approximates target function f

Two-Layer Networks are Universal Function Approximators

Theorem (Th 9 in CIML):

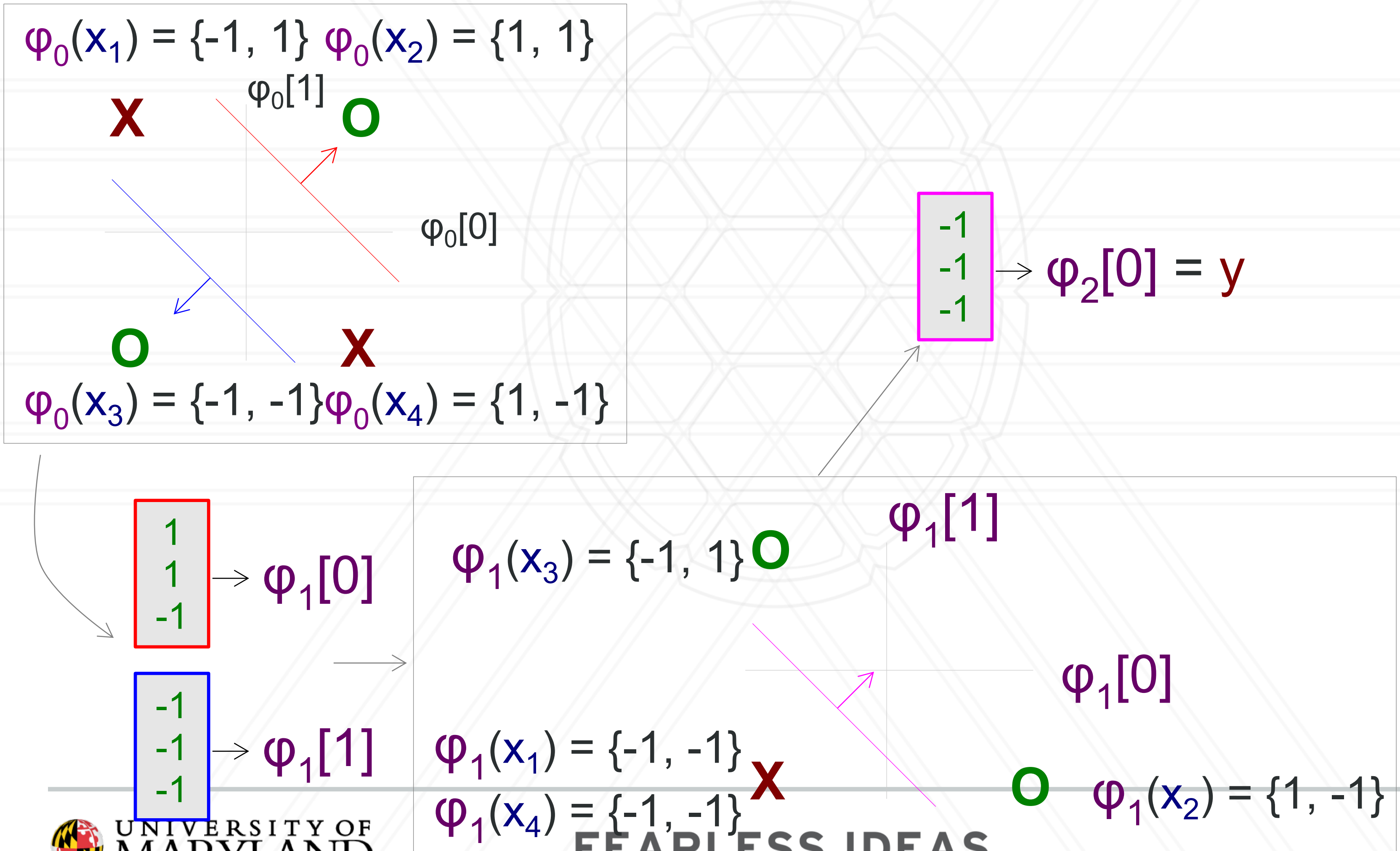
Let F be a continuous function on a bounded subset of D-dimensional space. Then there exists a two-layer neural network $\hat{F}$ with a finite number of hidden units that approximates F arbitrarily well. Namely, for all x in the domain of F, $\left|F(\boldsymbol{x}) - \hat{F}(\boldsymbol{x})\right| < \epsilon$

Example: a neural network to solve the XOR problem



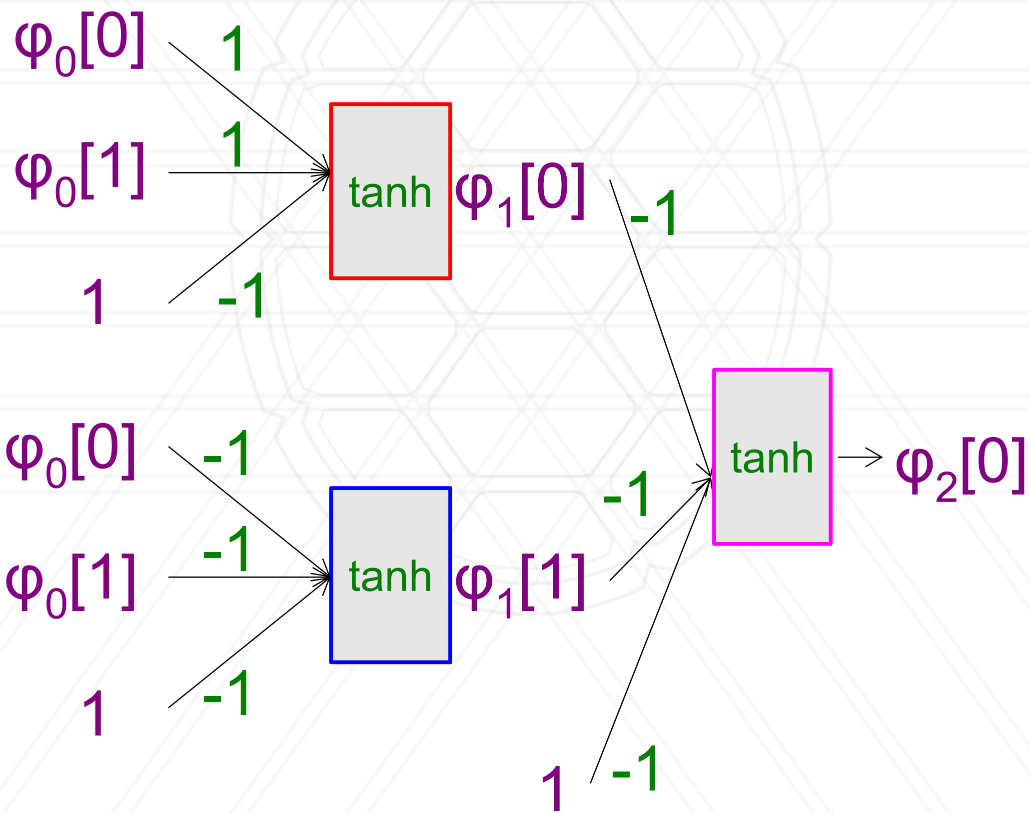
Example

- In new space, the examples are linearly separable!



Example

- The final net



Discussion

2-layer perceptron lets us

- Discover more complex decision boundaries than perceptron
- Learn combinations of features that are useful for classification

Key design question

- How many hidden units?
- More hidden units yield more complex functions
- Fewer hidden units requires fewer examples to train

Neural Networks

Today

- What are Neural Networks?
  - Multilayer perceptron
- How to make a prediction given an input?
  - Simple matrix operations + non-linearities
- Why are neural networks powerful?
  - Universal function approximators!

Next

- How to train them?

UNIVERSITY OF
MARYLAND

Furong Huang
3251 A.V. Williams, College Park, MD 20740
301.405.8010 / furongh@cs.umd.edu