CMSC 426, Computer Vision
Homework 0 - Answer Key

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1. Convolution of two Gaussian produces another Gaussian which is described by the following parameters: $\mu = \mu_1 + \mu_2$ and $\sigma = \sqrt{\sigma_1^2 + \sigma_2^2}$. That means mean is shifted and the distribution becomes more spread.

2. Yes they can. Consider $u = (2, 3)$, $v = (-1, 0)$, $w = (2, -6)$. In that case,

$$u.v = (2, 3).(-1, 0) = -2 + 0 = -2 < 0$$

$$v.w = (-1, 0).(2, -6) = -2 + 0 = -2 < 0$$

$$u.w = (2, 3).(2, -6) = 4 - 18 = -14 < 0$$

3. *Proof.* In that case, sum of the row vectors of $A - cI$ is 0 since it subtracts c from an element in each column. If we add a row to another one, the determinant does not change. So if we add first $n-1$ rows to the n^{th} row, all elements in the n^{th} row become 0. Therefore the determinant becomes 0 and c is an eigenvalue of A since it satisfies the characteristic equation $det(A - \lambda I) = 0$. ■

4. *Proof.* If λ is an eigenvalue of A and X is the corresponding eigenvector, we have $AX = \lambda X$. Therefore for any scalar s,

$$AX - sX = \lambda X - sX \implies (A - sI)X = (\lambda - s)X.$$

By the definition of eigenvalue and eigenvector, we can conclude that $\lambda - s$ is an eigenvalue of $A - sI$ for any scalar s and X is the corresponding eigenvector. ■

5. (a) True. We have $(AB)^T = B^T A^T = BA$. We need to show that A and A^T have the same eigenvalues. We know that

$$(A - \lambda I)^T = A^T - \lambda I$$

$$\implies det(A^T - \lambda I) = det((A - \lambda I)^T) = det(A - \lambda I)$$

since $det(A) = det(A^T)$.

(b) True. Suppose λ is an eigenvalue for AB with corresponding eigenvector v. So we have $\lambda Bv = B\lambda v = B(ABv) = (BA)Bv$. Thus λ is also an eigenvalue for BA with the corresponding eigenvector Bv. (Since B is invertible, Bv is not the zero vector unless $v = 0$.)

(c) False. Consider $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$. In that case B is invertible. Eigenvectors of AB are $(-0.7578, 0.6525)^T$ and $(-0.4031, -0.9151)^T$ while eigenvectors of BA are $(-0.8289, 0.5593)^T$ and $(-0.5949, -0.8038)^T$.

6. (a) Yes. If the rows of A are linearly independent, then $det(A) \neq 0$. In that case A is invertible, so $x = A^{-1}b$ is a solution. Another way to think of this is in such a case $rank(A) = m$, so column vectors of A span $\mathbb{R}^m$. Since $b \in \mathbb{R}^m$, it can be represented by the linear combination of column vectors of A.

   (b) No. If $m < n$, there will be infinitely many solutions since there will be free variables.

7. *Proof.* We have $Av = \lambda v \implies v = A^{-1}\lambda v \implies v = \lambda A^{-1} v \implies \frac{1}{\lambda} v = A^{-1} v$. ■