CMSC 426, Computer Vision
Homework 2 - Epipolar Geometry
Due on: 11:59:59PM on Thursday, April 12th

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Questions

1. In a stereo pair of images, corresponding 2D points x and x' that are generated from 3D coplanar points X that belong to a plane, π, can be related by a homography, $x' = H_\pi x$. These points can also be related via the fundamental matrix $x'Fx = 0$. Given coordinates of two corresponding pairs of points (x_1, x_1') and (x_2, x_2'), where $x_1' = H_{\pi_1} x_1$, $x_2' = H_{\pi_2} x_2$, $\pi_1 \neq \pi_2$, and given homographies H_{π_1} and H_{π_2}, write the set of equations that are used to compute F in terms of x_1, x_2, homographies and F itself. What is the degrees of freedom of F, and why not 9? (Note: Degrees of freedom means the number of independent parameters/elements of matrix.)

2. (a) Suppose F is the fundamental matrix of the pair of cameras (P, P'). What is the fundamental matrix for the pair (P', P).

(b) If for a point x in the first image, the corresponding epipolar line is Fx, then what is the epipolar line corresponding to x' in the second image?

(c) What are the left and right null-spaces of F? Give reasoning i.e. explain what null space is.

3. Imagine a camera whose projection matrix is defined as

$$\boldsymbol{M}_k = \boldsymbol{K}[\boldsymbol{R}|\boldsymbol{t}]$$

where $\boldsymbol{R} = \begin{pmatrix} r_1 & r_2 & r_3 \end{pmatrix}^T$ is the rotation matrix, $\boldsymbol{t} = \begin{pmatrix} t_x & t_y & t_z + k \end{pmatrix}^T$ is the translation vector, and K is the calibration matrix where

$$\boldsymbol{K} = \begin{bmatrix} kf & 0 & 0 \\ 0 & kf & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Note that as k increases, we increase both the focal length of the camera, and the Z distance between the camera and some point in the world by the same factor.
Let's consider the projection of some point $\boldsymbol{P}$.

(a) Express the image coordinates of the point P in homogeneous coordinates i.e. $(u, v, 1)$.

(b) Take the limit of u and v as k goes to infinity, and use L'Hopital's rule to express each limit as simply as possible.

(c) Construct a matrix $\boldsymbol{M}_\infty$ that has the same effect of $\boldsymbol{M}_k$ above, as $k \to \infty$. That is, we want the matrix to satisfy

$$\begin{bmatrix} \lim\limits_{k\to\infty} u \\ \lim\limits_{k\to\infty} v \\ 1 \end{bmatrix} = \boldsymbol{M}_\infty \begin{bmatrix} \boldsymbol{P} \\ 1 \end{bmatrix}.$$

This is called "orthographic projection."

Submission Guidelines

Answer the questions in a pdf file with the naming convention `YourDirectoryID_hw2.pdf` and submit them ELMS/Canvas. **YOU NEED TO TYPESET THE ANSWERS IN LaTeXor Word, HANDWRITTEN ANSWERS WILL BE GIVEN ZERO CREDIT! FEEL FREE TO DRAW DIAGRAMS BY HAND IF YOU WANT!**

Collaboration Policy

You are restricted to discuss the ideas with at most two other people. But the solutions you turn-in should be your own and if you **DO USE** (try not it and it is not permitted) other external solutions/solutions from other students - do cite them. For other honor code refer to the CMSC426 Spring 2018 website.