

Image Formation

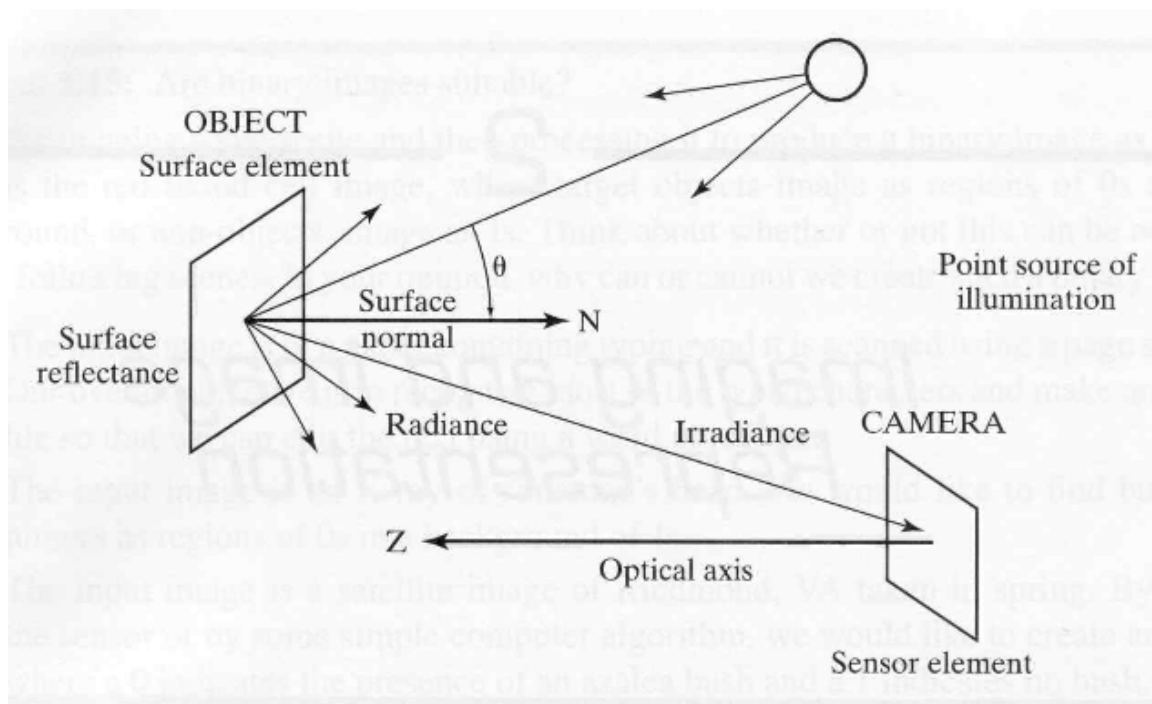
- **The two parts of the image formation process**

The geometry of image formation which determines where in the image plane the projection of a point in the scene will be located.

The physics of light which determines the brightness of a point in the image plane as a function of illumination and surface properties.

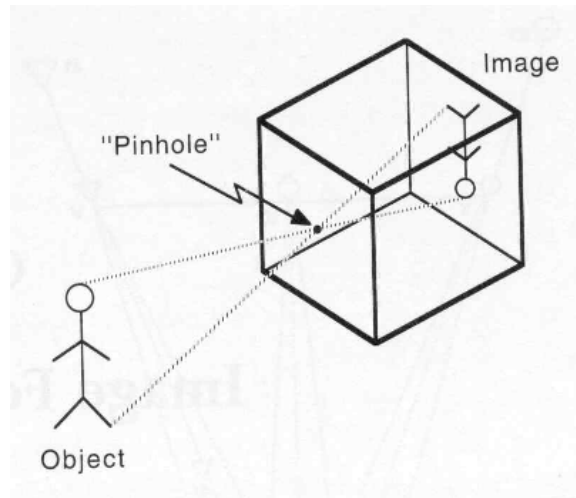
- **A simple model**

- The scene is illuminated by a single source.
- The scene reflects radiation towards the camera.
- The camera senses it via chemicals on film.



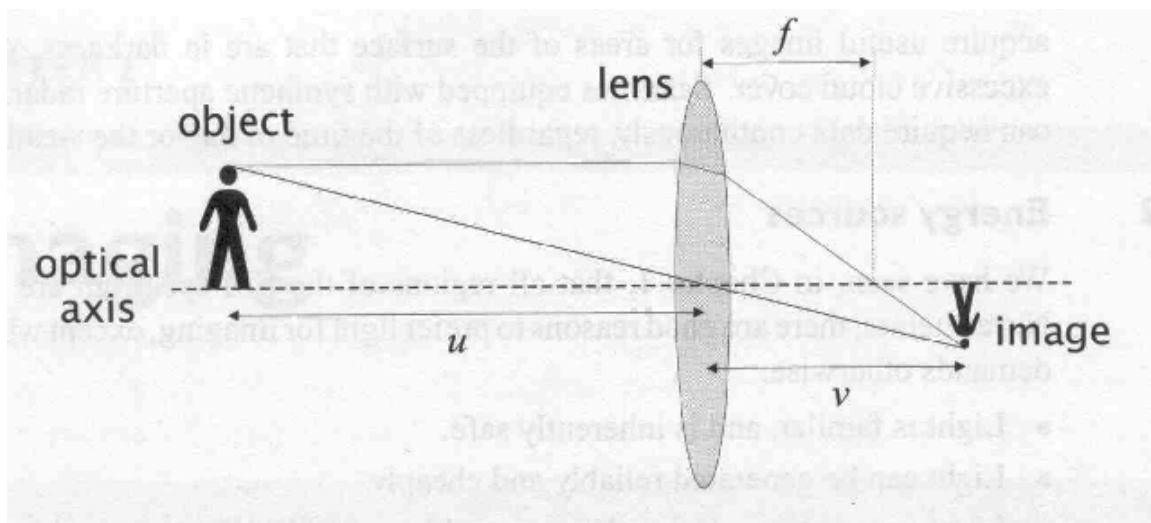
- **Camera Geometry**

- The simplest device to form an image of a 3D scene on a 2D surface is the "pinhole" camera.
- Rays of light pass through a "pinhole" and form an inverted image of the object on the image plane.

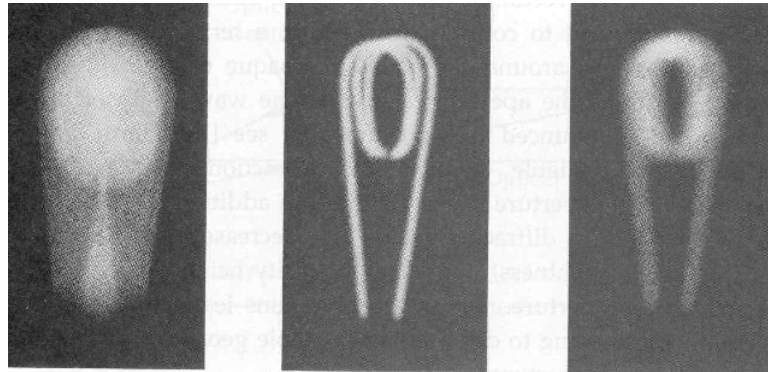


- **Camera Optics**

- In practice, the aperture must be larger to admit more light.
- Lens are placed in the aperture to focus the bundle of rays from each scene point onto the corresponding point in the image plane.



• Diffraction and Pinhole Optics



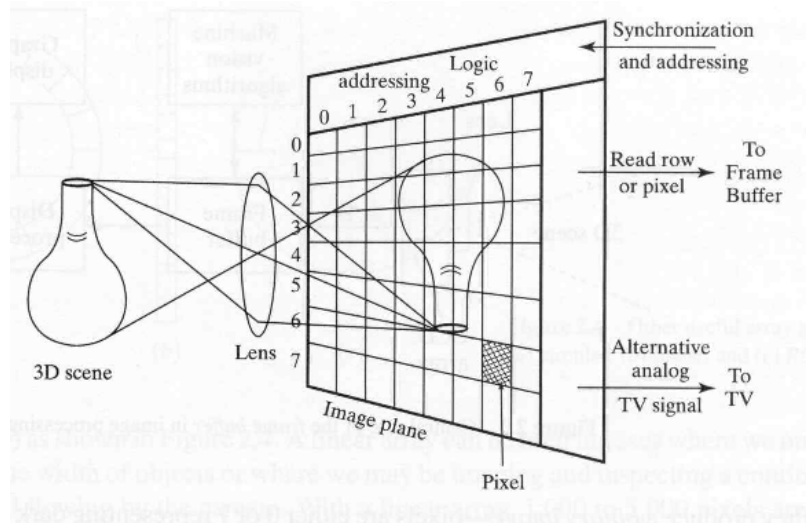
- If we use a wide pinhole, light from the source spreads across the image (i.e., not properly focused), making it blurry.
- If we narrow the pinhole, only a small amount of light is let in.
 - * the image sharpness is limited by diffraction.
 - * when light passes through a small aperture, it does not travel in a straight line.
 - * it is scattered in many direction (this is a quantum effect).
- In general, the aim of using lens is to duplicate the pinhole geometry without resorting to undesirable small apertures.

• Human Vision

- At high light levels, pupil (aperture) is small and blurring is due to diffraction.
- At low light levels, pupil is open and blurring is due to lens imperfections.

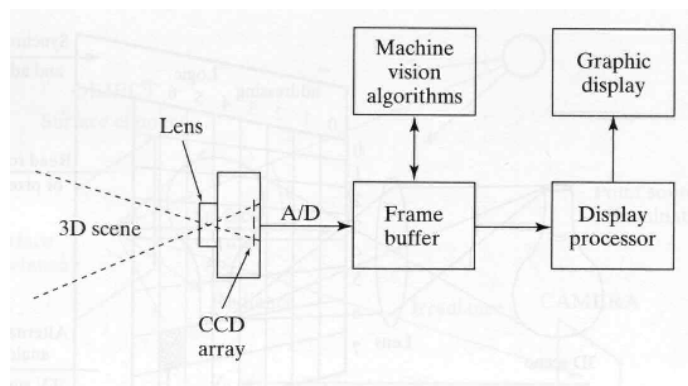
• CCD Cameras

- An array of tiny solid state cells convert light energy into electrical charge.
- Manufactured on chips typically measuring about 1cm x 1cm (for a 512x512 array, each element has a real width of roughly 0.001 cm).
- The output of a CCD array is a continuous electric signal (*video signal*) which is generated by scanning the photo-sensors in a given order (e.g., line by line) and reading out their voltages.



• Frame grabber

- The video signal is sent to an electronic device called the frame grabber.
- The frame grabber digitizes the signal into a 2D, rectangular array $N \times M$ of integer values, stored in the *frame buffer*.



CCD array and frame buffer

- In a CCD camera, the *physical image plane* is the CCD array of $n \times m$ rectangular grid of photo-sensors.
- The *pixel image plane* (frame buffer) is an array of $N \times M$ integer values (pixels).
- The position of the same point on the image plane will be different if measured in CCD elements (x, y) or image pixels (x_{im}, y_{im}) .
- In general, $n \neq N$ and $m \neq M$; assuming that the origin in both cases is the upper left corner we have:

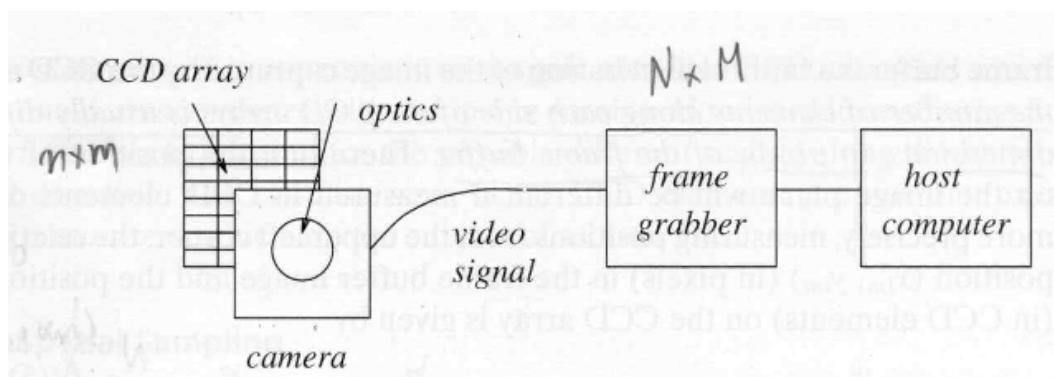
$$x_{im} = \frac{N}{n} x \qquad y_{im} = \frac{M}{m} y$$

where (x_{im}, y_{im}) are the coordinates of the point in the pixel plane and (x, y) are the coordinates of the point in the CCD plane.

- In general, it is convenient to assume that the CCD elements are always in one-to-one correspondence with the image pixels.
- Units in each case:

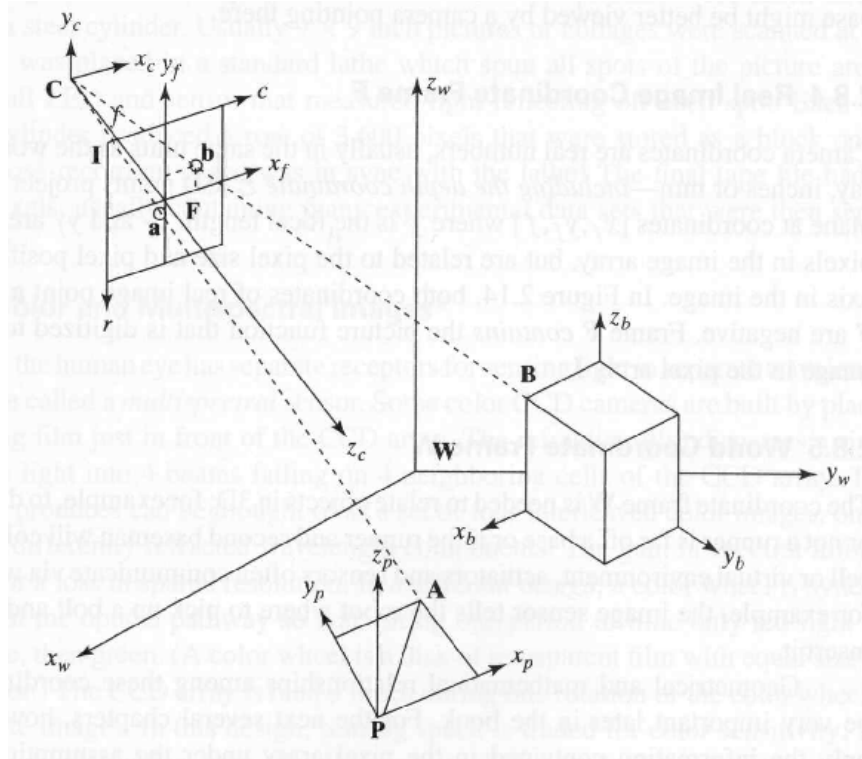
(x_{im}, y_{im}) is measured in pixels

(x, y) is measured, e.g., in millimeters.



Reference Frames

- Five reference frames are needed for general problems in 3D scene analysis.



• Object Coordinate Frame

- This is a 3D coordinate system: x_b, y_b, z_b
- It is used to model ideal objects in both computer graphics and computer vision.
- It is needed to inspect an object (e.g., to check if a particular hole is in proper position relative to other holes)
- The coordinates of 3D point B , e.g., relative to the object reference frame are $(x_b, 0, z_b)$
- Object coordinates do not change regardless how the object is placed in the scene.

$$\text{Notation: } (X_o, Y_o, Z_o)^T$$

• World Coordinate Frame

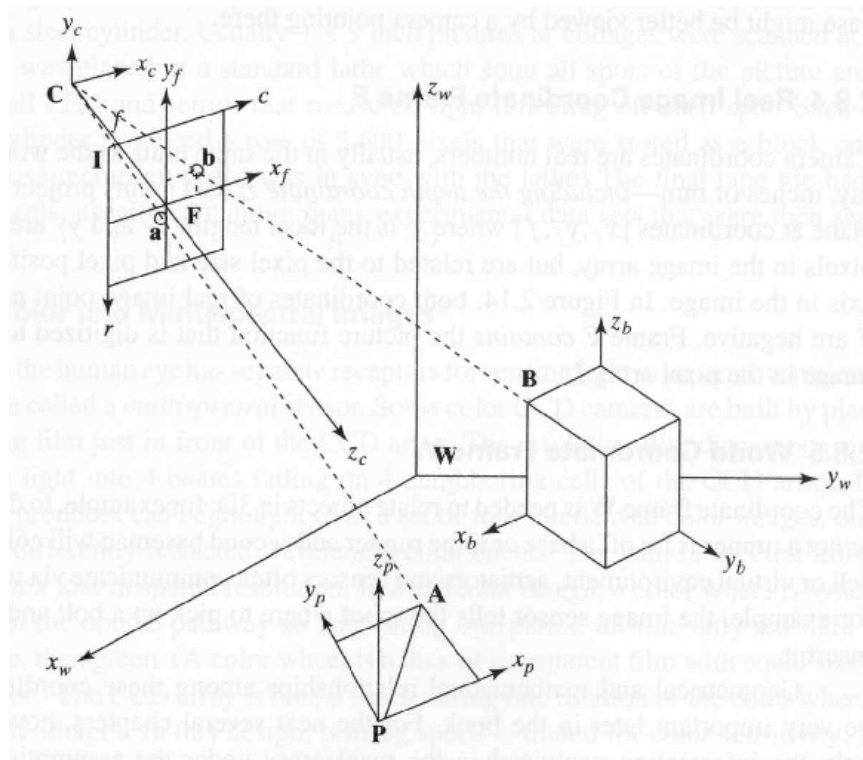
- This is a 3D coordinate system: x_w, y_w, z_w
- The scene consists of object models that have been placed (rotated and translated) into the scene, yielding object coordinates in the world coordinate system.
- It is needed to relate objects in 3D (e.g., the image sensor tells the robot where to pick up a bolt and in which hole to insert it).

$$\text{Notation: } (X_w, Y_w, Z_w)^T$$

• Camera Coordinate Frame

- This is a 3D coordinate system (x_c, y_c, z_c axes)
- Its purpose is to represent objects with respect to the location of the camera.

$$\text{Notation: } (X_c, Y_c, Z_c)^T$$



- **Image Plane Coordinate Frame (CCD plane)**

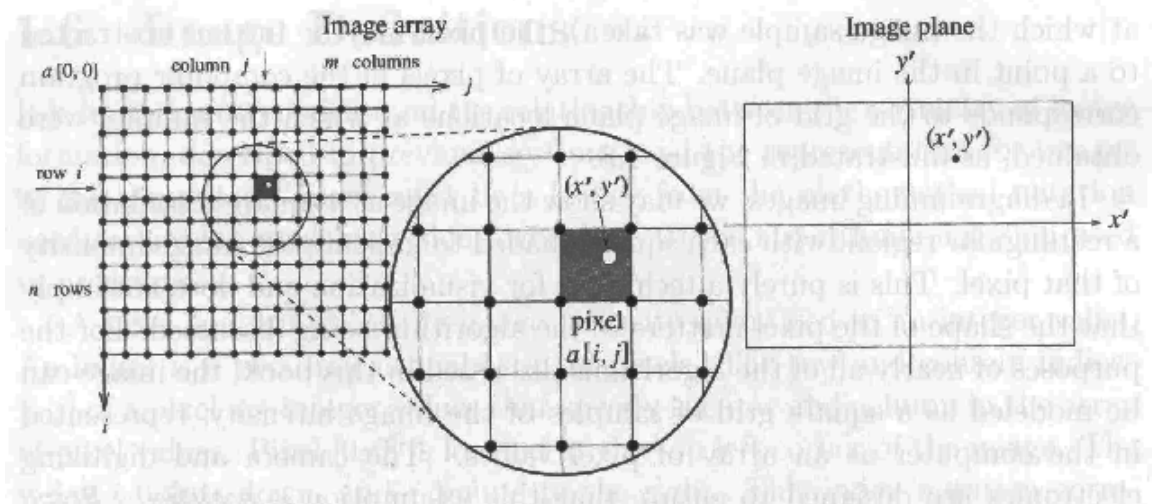
- This is a 2D coordinate system (x_f, y_f axes)
- Describes the coordinates of 3D points projected on the image plane.
- The projection of A , e.g., is point a whose both coordinates are negative.

Notation: $(x, y)^T$

- **Pixel Coordinate Frame**

- This is a 2D coordinate system (r, c axes)
- Each pixel in this frame has an integer pixel coordinates.
- Point A , e.g., gets projected to image point (a_r, a_c) where a_r and a_c are integer row and column.

Notation: $(x_{im}, y_{im})^T$



- **Transformations between frames**

