

# APPLIED MECHANISM DESIGN FOR SOCIAL GOOD

**JOHN P DICKERSON**

Lecture #17 – 3/29/2018

**CMSC828M**  
**Tuesdays & Thursdays**  
**9:30am – 10:45am**



**COMPUTER SCIENCE**  
UNIVERSITY OF MARYLAND

**THIS CLASS:**

**COMBINATORIAL ASSIGNMENT  
PROBLEMS & COURSE MATCH**

*PART I: JOHN DICKERSON*

*PART II: JANIT ANJARIA*

# RECALL: DRF

Proportional demands (a.k.a. Leontief preferences)

$$u(x_1, \dots, x_m) = \min \left\{ \frac{x_1}{w_1}, \dots, \frac{x_m}{w_m} \right\}$$

**Dominant resource:** resource the agent has the biggest share of out of all resources available:

- 16 CPUs, 10 GB available, user allocated 4 CPUs, 8 GB
- Dominant resource is GB, because  $4/16 \text{ CPU} < 8/10 \text{ GPU}$

**Dominant share:** fraction of dominant resource allocated

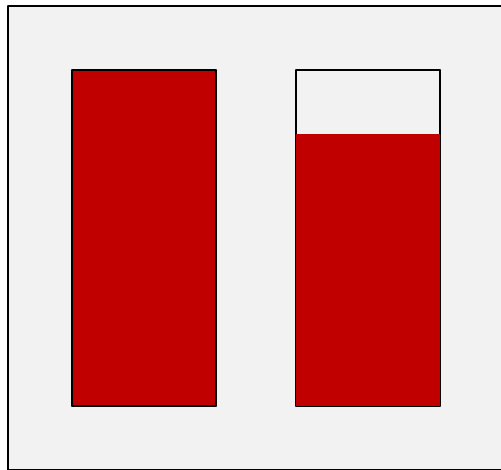
- Above, dominant share is  $8/10 = 80\%$

**DRF:** application of max-min fairness to dominant shares

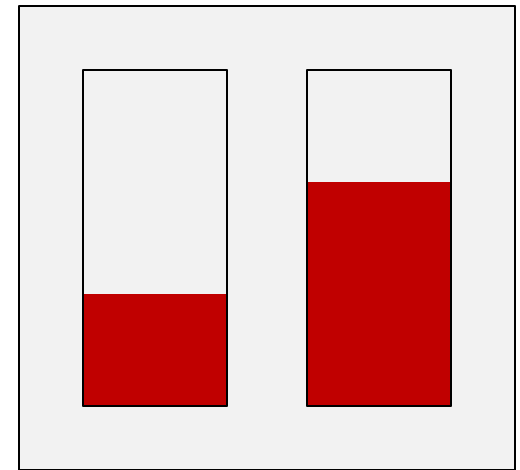
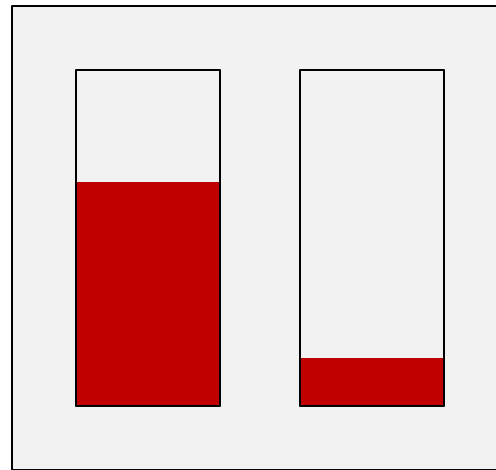
- Equalize the dominant share amongst agents

# STATIC DRF MECHANISM

Dominant Resource Fairness = equalize largest shares  
(a.k.a. dominant shares)



Total



# ALTERNATIVE: MAKE A MARKET

## Competitive Equilibrium from Equal Incomes (CEEI):

- **Agents report their preferences over sets of items**
- **Give agents an equal budget of funny money**
- **Computer finds prices that clear the market**
  - That is, prices such that when each agent chooses its most favored set that it can afford, the market clears
- **Assign all resources to agents based on their demands and these computed prices**

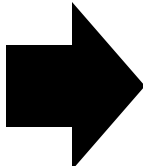
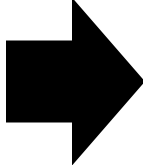
# CEEI EXAMPLE: DIVISIBLE RESOURCES

Supply: {1 **cake**, 1 **doughnut**}

Two agents, both with \$1 (funny money), capacity of 1

- **A**: **cake** = 1/2, **doughnut** = 1
- **B**: **cake** = 1/4, **doughnut** = 1

Market clearing prices: **cake** = \$2/5, **doughnut** = \$8/5

- **A** wants to max  $1/2c + 1d$   
s.t.  $c + d \leq 1$   
 $p_c c + p_d d \leq 1$   ???????????  
Max: 1/2 cake, 1/2 doughnut
- **B** wants to max  $1/4c + 1d$   
s.t.  $c + d \leq 1$   
 $p_c c + p_d d \leq 1$   Max: 1/2 cake, 1/2 doughnut  
(and many others – clearinghouse chooses!)

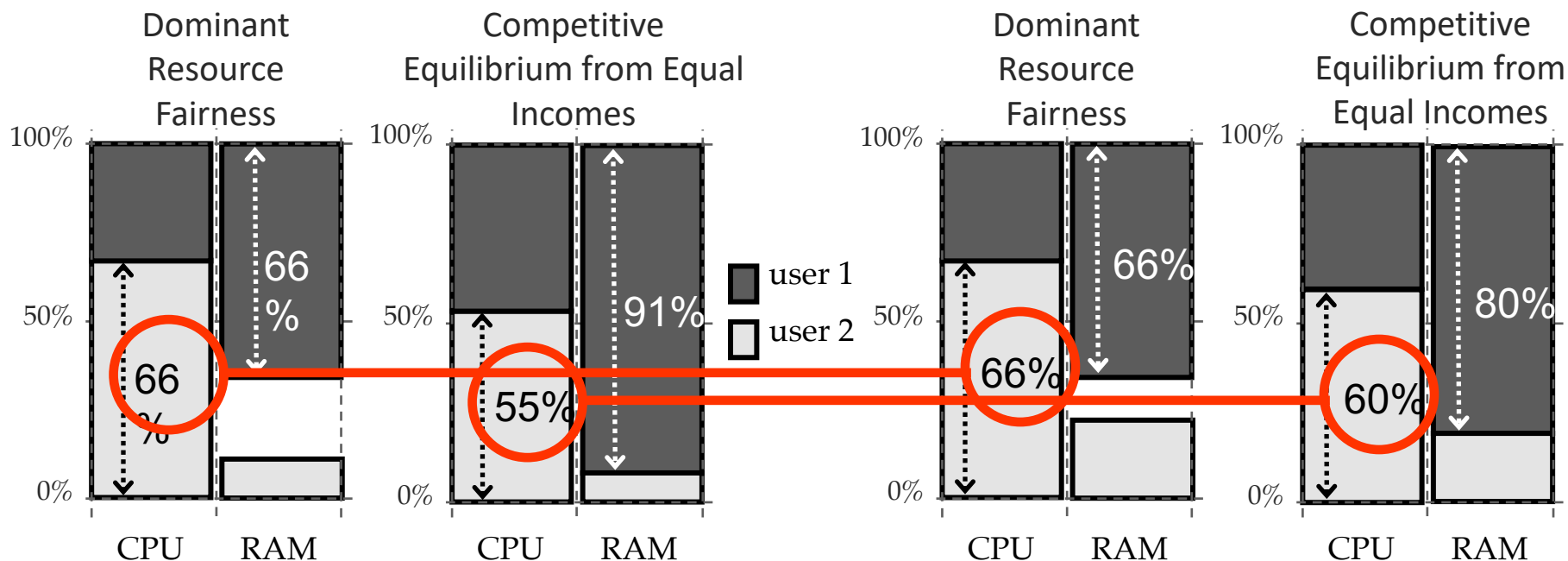
# CEEI PROPERTIES

- **Envy-free** ??????????
  - **Yes!** Given the prices, you bought the best bundle you could afford
  - If you envy somebody else's bundle, you could've purchased it!
- **Pareto-efficient** ??????????
  - **Yes!** Market is cleared → taking a Pareto step involves taking a resource from one agent and giving it to somebody new ... but this lowers their utility by above
- **Strategy proof** ??????????
  - **No!** Intuition: CEEI clears the market → can game the system by requesting more underutilized resources

# DRF VS CEEI

A1: <1 CPU, 4 GB> A2: <3 CPU, 1 GB>

- DRF more fair, CEEI better utilization



A1: <1 CPU, 4 GB> A2: <3 CPU, 2 GB>

- A2 increased her share of both CPU and memory



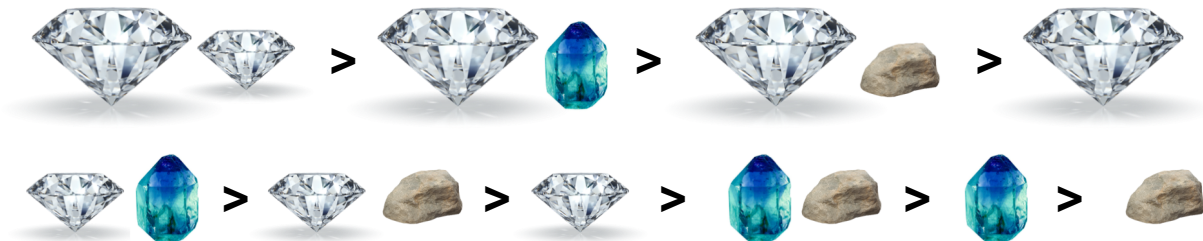
# CEEI FOR INDIVISIBLE ITEMS?



Two agents

Capacity: 2

Both agents will share the same preference profile:



Market clearing prices ??????????

- **Don't exist!** For any price, for any item, either both agents demand that item or both do not.
- Small changes in price can cause big changes in demand

# APPROXIMATE-CEEI

Can we tiebreak somehow?

Idea: give agents slightly different, but **roughly equal budgets**

- For each agent, draw budget from  $[1, 1 + B)$
- $0 < B < \min(1/m, 1/(k-1))$  –  $k$  is capacity of agent
- Note: if  $B = 0$ , this is just CEEI

Still “feels fair” – random winners and losers in the budget draw, and the playing ground is still roughly equal.

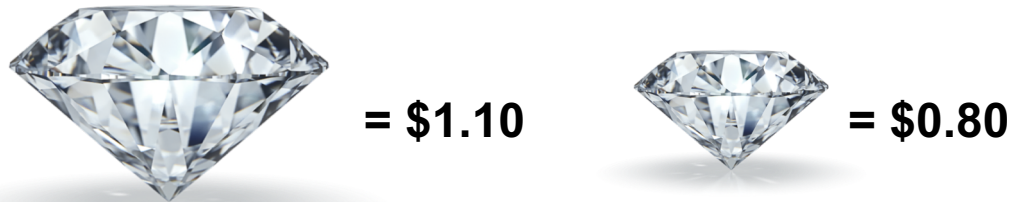
# A-CEEI FOR INDIVISIBLE ITEMS

Two agents

Capacity: 2

Agent 1's budget: \$1.2

Agent 2's budget: \$1



??????????????



# A-CEEI: PROPERTIES

Always exists if  $B > 0$  (need unequal budgets)

The market **approximately clears**:

- There exist prices that clear the market to within an error of at most  $\sqrt{k \cdot m/2}$   
????????????????
- Error does not depend on the number of participants  $\rightarrow$  error goes to zero as a fraction of the underlying endowment

Approximately **strategy proof**

- “Strategy-proof in the large”

Bounded **envy free**

Very **difficult to compute!**

**NEXT UP:  
COURSE MATCH**

*PRESENTER: JANIT ANJARIA*



**Wharton**  
UNIVERSITY of PENNSYLVANIA

**NEXT CLASS:  
INCENTIVE AUCTIONS & SPECTRUM  
REPACKING**

