School Choice: A Mechanism

Design Approach

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Presented By:

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Before school choice: Wealthy
Before school choice: Not wealthy
Why school choice?
Your Choice  
It Should Be  

Your Kids  
They're  

Tax Money  
It's Your  

SCHOOL CHOICE
Why do we need an assignment mechanism?

The design of a student assignment mechanism is a central issue in school choice. It is not possible to assign each student to her top choice school, a central problem in school choice.
**Problem setup:**

**Input:**

- Number of students.
- Each student is to be assigned one seat at a school.
- Each school has a maximum capacity but there is no shortage in total number of seats.
- Each student has a strict priority ordering of all students.
- Each school has a strict priority preference.

**Output:**

- Each student is to be assigned one seat at a school.
Problem setup: School Priority Ordering

School Priority ordering?!?

Here, priorities do not represent school preferences but they are imposed by state or local laws, the priority ordering of a student can be different at different schools!

Here, priorities do not represent school preferences but they are imposed by state

● Students who live in the attendance area of a school must be given priority over students who do not live in the school's attendance area.

● Students requiring a bilingual program must be given priority in schools that offer such programs.

● Siblings of students already attending a school must be given priority.

● Siblings of students already attending a school must be given priority.
Problem setup:

Output:

- Assignment of students to schools.
- Each student is assigned to exactly one school.
- No school is assigned to more students than its capacity.
Problem setup:

Goals:

- Envy free
- Strategy-proof
- Pareto efficient

An optimal mechanism should be:
Boston Student Assignment Mechanism
Boston Student Assignment Mechanism

1. Each student submits a preference ranking of the schools.

2. For each school a priority ordering is determined according to the following hierarchy:
   - First priority: sibling and walk zone.
   - Second priority: sibling.
   - Third priority: walk zone.
   - Fourth priority: other students.

3. Students in the same priority group are ordered based on a previously announced lottery.
The final phase is the student assignment based on preferences and priorities:

- **Round 1:** Only the first choices of the students are considered.
- **Round 2:** Only the second choices of these students are considered.
- **Round k:** Only the k-th students are considered.
Boston student assignment mechanism is not strategy-proof. Even if a student has very high priority at school and instead lists their number two school first, they may try to avoid "wasting" their first choice. It may be optimal for some families to be strategic in listing their school choices. If a parent thinks their favorite school is oversubscribed and they have a close second favorite, they may try to avoid "wasting" their first choice. Even if a student has very high priority at school, unless she lists it as her top choice, she loses her priority to students who have listed it as their top choice. Boston student assignment mechanism is not strategy-proof.
Boston Student Assignment Mechanism
Priority among applicants is determined by a random lottery. Lottery office

Student

Apply to 3 schools

Lottery office

Remaining applications are put on a waiting list. Lottery office

If she accepts an offer, she is assigned a seat. If she declines an offer, Lottery office

3 days to accept or decline an offer. Student

If seats become available, offers are made to students on the waiting list. Lottery office

Available seats are offered to students with the highest priority. Lottery office

Random lottery is determined by a Lottery office
The optimal application strategy of students is unclear under the Columbus student assignment mechanism.

When a family gets an offer from its second or third choice, it is unclear whether the optimal strategy is declining this offer or accepting it.

Another major difficulty with the Columbus student assignment mechanism concerns efficiency: Consider two students, each of whom hold an offer from the other’s first choice. Since they do not know whether they will receive better offers, they may as well accept these offers, and this in turn yields an inefficient matching.

The optimal application strategy of students is unclear under the Columbus mechanism.
Mechanism

Columbus Student Assignment
Student assignment mechanisms:

Dormitory Rooms: Random serial dictatorship
Student assignment mechanisms:

- Dormitory rooms: Random serial dictatorship
  - Order the students with a lottery and assign the first student her top choice, the next student her top choice among the remaining slots, and so on.

- This mechanism is not only Pareto efficient, but also strategy-proof.
Student assignment mechanisms:

- 
  Dormitory rooms: Random serial dictatorship
  - A single lottery cannot be used to allocate school seats to students.
  - It is this school-specific priority feature that complicates the student assignment process.
  - A student assignment mechanism should be flexible enough to give students different priorities at different schools.

Dormitory rooms: Random serial dictatorship
Student assignment mechanisms:
Dormitory Rooms: Random Serial
Dictatorship
College Admissions

Student assignment mechanisms:
Student assignment mechanisms:

- College Admissions

● The central difference between the college admissions and school choice is that in college admissions, schools themselves are agents which have preferences over students.

● Whereas in school choice, schools are merely "objects" to be consumed by the students.

A student should not be rejected by a school because of her personality or ability level.
College Admissions:

Student-college pair \((i, s)\) where student \(i\) prefers college \(s\) to her assignment and college \(s\) prefers student \(i\) to one or more of its admitted students.

School choice: Student-school pair \((i, s)\) where student \(i\) prefers school \(s\) to her assignment and she has higher priority than some other student who is assigned a seat at school \(s\).

College Admissions:

Student assignment mechanisms:

School choice: Student-school pair \((i, s)\) where student \(i\) prefers school \(s\) to her assignment and her priority is higher than some other student who is assigned a seat at school \(s\).
Student assignment mechanisms:

Gale and Shapley context of college admissions

- A stable matching in the context of college admissions eliminates justified envy in the context of school choice.
- Good news: There exists a stable matching which is preferred to any stable matching by every student in the context of college admissions.

Gale and Shapley context of college admissions

College Admissions
Step 1:

Gale-Shapley Student Optimal Stable Mechanism

1. Each student proposes to her first choice.
2. Each school tentatively assigns its seats to its proposers one at a time following their priority order.
3. Any remaining proposers are rejected.
4. Each student proposes to her first choice.
Each student is assigned her final tentative assignment.

The algorithm terminates when no student proposal is rejected and
any remaining proposers are rejected.

Each school considers the students it has been holding together
with its new proposers and tentatively assigns its seats to these
students one at a time following their priority order.

In general, Step k:

Gale-Shapley Student Optimal Stable Mechanism
Good news:

- It is strategy-proof.
- It Pareto-dominates any other mechanism that eliminates justified envy.

Gale-Shapley Student Optimal Stable Mechanism
Gale-Shapley Student Optimal Stable Mechanism

**Good news:**
- It is strategy-proof.
- It Pareto-dominates any other mechanism that eliminates justified envy.

**Bad news:**
- There is a potential trade-off between stability and Pareto efficiency.

**Good news:**
- It is strategy-proof.
Example:

There are three students $i_1$, $i_2$, $i_3$, and three schools $s_1$, $s_2$, $s_3$, each of which has only one seat.

The priorities of schools and the preferences of students are as follows:

- $s_1 : i_3, i_2, i_1$
- $s_2 : i_2, i_3, i_1$
- $s_3 : i_1, i_3, i_2$

Gale-Shapley Student Optimal Stable Mechanism
Example:

Gale-Shapley Student Optimal Stable Mechanism
Gale-Shapley Student Optimal Stable Mechanism

Example:

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Gale-Shapley Student Optimal Stable Mechanism
Gale-Shapley Student Optimal Stable Mechanism

Example:

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- \( \varepsilon \) \( s \) 
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stable mechanism
Example: Gale-Shapley Student Optimal Stable Mechanism
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Gale-Shapley Student Optimal Stable Mechanism
Example:

Gale-Shapley Student Optimal Stable Mechanism

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But this matching is Pareto-dominated by

Example:

Gale-Shapley Student Optimal Stable Mechanism
Top trading cycles mechanism
Top trading cycles mechanism

A competing mechanism which is Pareto efficient but which does not completely eliminate justified envy.

Suppose that if student $i_1$ has higher priority than student $i_2$, then she has a better opportunity to get into school $s$, but that does not necessarily mean that she is entitled a seat at school $s$ before student $i_2$. It rather represents the opportunity to get into school $s$ that does not necessarily mean that she is entitled a seat at school $s$.

- If $i_1$ has higher priority than $i_2$, then she has a better opportunity to get into school $s$.

- A competing mechanism which is Pareto efficient but which does not completely eliminate justified envy.
Step 1:

Top trading cycles mechanism

Zero, the school is also removed.

The counter of each school in a cycle is reduced by one and if it reduces to
removed.

Every student in a cycle is assigned a seat at the school she points to and is

There is at least one cycle.

Each school points to the student who has the highest priority for the school.

Each student points to her favorite school under her announced preferences.

Still available at the school.

Assign a counter for each school which keeps track of how many seats are

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Assign a counter for each school which keeps track of how many seats are
Top trading cycles mechanism

In general Step k:

- Each remaining student points to her favorite school among the remaining schools.
- Each remaining school points to the student with highest priority among the remaining students.
- There is at least one cycle.
- Every student in a cycle is assigned a seat at the school that she points to.
- The counter of each school in a cycle is reduced by one and if it reduces to zero the school is also removed.
- Each remaining student points to her favorite school among the remaining schools.
- Each remaining student points to her favorite school among the remaining students.
Top trading cycles mechanism

Example:

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The priorities of the schools and the preferences of the students are as follows:

- There are eight students i₁, ..., i₈ and four schools s₁, s₂, s₃, s₄ have three seats each.
- Schools s₁, s₂ have two seats each.

Top trading cycles mechanism
Top trading cycles mechanism

Step 1: Output
Top trading cycles mechanism

Step 2:

Output

\[ z^T_s = z, \ s \in S, \ s_1^Z = 0, \ s_2^Z = 1, \ s_3^Z = 2, \ s_4^Z = 2 \]
Top trading cycles mechanism

Step 3:

$z_4 = 4$, $s_4 = 0$, $s_3 = 0$, $s_2 = 0$, $s_1 = 0$

Output:

$s_4 = 4$, $s_3 = 3$, $s_2 = 2$, $s_1 = 0$, $s_2 = 0$, $s_3 = 1$, $s_4 = 2$
Step 4: Top trading cycles mechanism

\[ s_1 = s_2 = 0, s_3 = 1, s_4 = 1 \]

Output:

\[ i_8 = s_4 \]
Good news:

- It is strategy-proof.
- It eliminates justified envy.
- It Pareto-dominates any other mechanism that eliminates justified envy.

Top trading cycles mechanism
Which Mechanism Shall Be Chosen?
Which Mechanism Shall Be Chosen?

Both mechanisms are strategy-proof, so the choice between them depends on the structure and interpretation of the priorities.
Which Mechanism Shall Be Chosen?

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It depends on the application
Which Mechanism Shall Be Chosen?

Both mechanisms are strategy-proof, so the choice between them depends on the structure and interpretation of the priorities. It depends on the application.

In other applications, the top trading cycles mechanism may be more appealing. In some applications, policy makers may rank complete elimination of justified envy before full efficiency, and Gale-Shapley student optimal stable mechanism can be used in those cases.
Controlled choice mechanism
One of the major concerns about the implementation of school choice plans is that they may result in racial and ethnic segregation at schools. Because of these concerns, choice plans in some districts are limited by court ordered desegregation guidelines. Both Gale-Shapley student optimal stable mechanisms and the top trading cycles mechanism can be easily modified to accommodate controlled choice constraints by imposing type-specific quotas.
Controlled choice mechanism

- Suppose that there are different types of students and each student belongs to one type.
- If the controlled choice constraints are perfectly rigid then there is no need to modify the mechanisms.
- For each type of students, one can separately implement the mechanism in order to allocate the seats that are reserved exclusively for that type.
- When the controlled choice constraints are flexible, modification are need to both Gale-Shapley student optimal stable and top trading cycles mechanisms.
Step K:

With Type-Specific Quotas

- Gale-Shapley Student Optimal Stable Mechanism

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Step K:

- Each student who was rejected in the previous step proposes to her next choice.
- Each school considers the students it has been holding together with its new choice.
- Any remaining proposers are rejected.
- If the quota of a type fills, the remaining proposers of that type are rejected and the tentative assignment proceeds with the students of the other types.
- Following their priority order, proposers and tentatively assigns its seats to these students one at a time.
This modified mechanism satisfies the following version of the fairness requirement:

1. Students $i$ and $j$ are of different types assigned a seat at school $s$, then:

2. The quota for the type of student $i$ is full at school $s$.

If there is an unmatched student-school pair $(i,s)$ where student $i$ prefers school $s$ to her assignment and she has higher priority than some other student who is assigned a seat at school $s$, then:

Gale-Shapley Student Optimal Stable Mechanism with Type-Specific Quotas
Step K:

- Each remaining student points to her favorite remaining school among those which have room for her.
- Each remaining school points to the student with the highest priority among remaining students.
- There is at least one cycle.
- Every student in a cycle is assigned a seat at the school that she points to and is removed.
- The associated type-specific counter is reduced by one as well.
- All other counters stay put.
- In case the counter of a school reduces to zero, the school is removed.
- If there is at least one remaining student, then we proceed with the next step.
- The counter of each school in a cycle is reduced by one and depending on the student it is assigned to.

Quotas

Top Trading Cycles Mechanism with Type-Specific Quotas