CMSC 250
Discrete Structures
Predicate Logic
Propositional logic has limitations

**Definition**

All men are mortal
Socrates is a man
\[ \therefore \text{Socrates is mortal} \]
Predicates

Definition
A *Predicate* is a sentence (containing variables) that is either true or false depending on the values substituted for the variables.

Example
Karim is a student at University of Maryland.
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Karim is a student at University of Maryland.

The word “Karim” is a subject and “is a student at University of Maryland” is a predicate.
Predicates in logic

Definition
To obtain predicate, we can remove one or more nouns

Example
for the sentence,
Karim is a student at University of Maryland.
Let,

P: is a student at University of Maryland
Q: is a student at
Both P and Q are predicate symbols.
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# Predicates in logic

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## Predicate Symbols

### Example

<table>
<thead>
<tr>
<th>Predicate</th>
<th>Domain (set)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q(x): x is even</td>
<td>x ∈ ℤ</td>
</tr>
<tr>
<td>P(x, y, z): $x^2 + y^2 = z^2$</td>
<td>x, y, z ∈ ℜ</td>
</tr>
<tr>
<td>R(a, b): a is a factor of b</td>
<td>a, b ∈ ℤ</td>
</tr>
<tr>
<td>S(a, b): a is taller than b</td>
<td>a, b ∈ students in this class</td>
</tr>
<tr>
<td>G(Y): y is green</td>
<td>y ∈ M&amp;M</td>
</tr>
<tr>
<td>A(c): c is under attack</td>
<td>c ∈ Computers</td>
</tr>
</tbody>
</table>
Logical Connectives

**Definition**

The logical connectives can be used to join predicates to make more complex predicates.

**Examples**

- \( P(x) = \sim Q(x) \lor R(x) \)
- \( T(x,y) = (A(x) \land G(x,y)) \rightarrow \sim L(y) \)
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Quantifiers

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- We bind the variables using quantifiers, to find out,
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- We have to be able to figure out the truth or falsehood of statements that include variables.
- We bind the variables using quantifiers, to find out,
  - Whether the claim applies to all values of the variable - universal quantification
  - whether it may only apply to some - existential quantification.
Universal Quantifier

Definition

The *Universal quantifier*, \( \forall \) (for all), says that a statement must be true for all values of a variable.

Example

- All humans are mortal
  \[ \forall x: \text{Human}(x) \rightarrow \text{Mortal}(x) \]
- If \( x \) is positive then \( x + 1 \) is positive
  \[ \forall x : x > 0 \rightarrow x + 1 > 0 \]
**More Universal Quantifier**

**Definition**

To make the universe of the values explicit, we use set membership notation.

**Example**

\[
\forall x \in \mathbb{Z} : x > 0 \rightarrow x + 1 > 0
\]

\[
\equiv \forall x : x \in \mathbb{Z} \rightarrow (x > 0 \rightarrow x + 1 > 0)
\]

\[
\equiv \forall x : (x \in \mathbb{Z} \land x > 0) \rightarrow x + 1 > 0
\]

\[
\forall x : P(x) \equiv \text{to a very large AND}
\]

**Example**

\[
\forall x \in \mathbb{N} : P(x)
\]

\[
P(0) \land P(1) \land P(2) \land \ldots
\]
Existential Quantifier

**Definition**

The *Existential quantifier*, \( \exists \) (there exists), says that a statement must be true for at least one value of the variable.

**Example**

There is a student in CMSC 250
\[ \exists x \in P \text{ such that } x \text{ is a student in CMSC 250} \]
where \( P \) is a set of all people.
Existential Quantifier

Example

\[ \exists x \in \mathbb{Z}: x = x^2 \]

Definition

Therefore, if \( Q(x) \) is a predicate and \( D \) the domain of \( x \), an existential statement has the form \( \exists x \in D \) such that \( Q(x) \).
Negation and Quantifier

**Definition**

The following equivalencies hold:

\[ \neg \forall x: P(x) \equiv \exists x: \neg P(x) \]
\[ \neg \exists x: P(x) \equiv \forall x: \neg P(x) \]

Quantifier version of De Morgan’s laws
Negation and Quantifier

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Quantifier version of De Morgan’s laws

Example

- not all humans are mortal is equivalent to finding some human that is not mortal.
- No human is mortal is equivalent to showing that all humans are not mortal.
Translation Examples

Example

- All crows are black
Translation Examples

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\[ \forall x : \text{Crow}(x) \rightarrow \text{Black}(x) \]
Translation Examples

Example

- All crows are black
  \[ \forall x : \text{Crow}(x) \rightarrow \text{Black}(x) \]
- Some cows are brown
  \[ \exists x : \text{Cow}(x) \land \text{Brown}(x) \]
- No cows are blue
  \[ \neg \exists x : \text{Cow}(x) \land \text{Blue}(x) \]
- All that glitters is not gold
  \[ \neg \forall x : \text{Glitters}(x) \rightarrow \text{Gold}(x) \]
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- No shirt, no service
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- No shirt, no service: $\forall x: \neg \text{Shirt}(x) \rightarrow \neg \text{Served}(x)$
- Every event has a cause
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- No shirt, no service $\forall x: \neg\text{Shirt}(x) \rightarrow \neg\text{Served}(x)$

- Every event has a cause $\forall x \exists y: \text{Causes}(y, x)$

- Every even number greater than 2 can be expressed as the sum of two primes.

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- No shirt, no service \( \forall x: \neg \text{Shirt}(x) \rightarrow \neg \text{Served}(x) \)

- Every event has a cause \( \forall x \exists y: \text{Causes}(y,x) \)

- Every even number greater than 2 can be expressed as the sum of two primes.
  \( \forall x : (\text{Even}(x) \land x > 2 \rightarrow \exists p \exists q: \text{Prime}(p) \land \text{Prime}(q) \land (x = p + q)) \)