CMSC 250
Discrete Structures
Sequences, Summations and Products
Practice finding an explicit formula

Figure out the formula for this sequence:

$$1, -\frac{1}{4}, \frac{1}{9}, -\frac{1}{16}, \frac{1}{25}, \ldots$$
Summation & product

- Sum of specified items
  \[ \sum_{k=1}^{6} 2^k = 2^1 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6 \]

- Product of specified items
  \[ \prod_{k=1}^{5} 2k = 2(1) \times 2(2) \times 2(3) \times 2(4) \times 2(5) \]
Variable ending point

- $n$ as the index of the final term

\[
\sum_{k=0}^{n} \frac{k + 1}{n + k}
\]

Evaluate for
- $n = 2$
- $n = 3$
Nesting of sum/product

- Variations (same or different ?)

\[
\sum_{j=1}^{n} \sum_{i=1}^{m_j} Y_{ij}^2 = \sum_{j=1}^{n} \left( \sum_{i=1}^{m_j} Y_{ij} \right)^2 = \left( \sum_{j=1}^{n} \sum_{i=1}^{m_j} Y_{ij} \right)^2
\]
Telescoping series

\[ \sum_{k=1}^{n} \left( \frac{k}{k+1} - \frac{k+1}{k+2} \right) \]

\[ \prod_{i=1}^{n} \left( \frac{i}{i+1} \right) \]
Merging and Splitting

- **Summations**

\[ \sum_{k=m}^{n} a_k + \sum_{k=m}^{n} b_k = \sum_{k=m}^{n} (a_k + b_k) \]

\[ \sum_{k=m}^{n} a_k = \sum_{k=m}^{i} a_k + \sum_{k=i+1}^{n} a_k \]

- **Products**

\[ \prod_{k=m}^{n} a_k \times \prod_{k=m}^{n} b_k = \prod_{k=m}^{n} (a_k \times b_k) \]

\[ \prod_{k=m}^{n} a_k = \prod_{k=m}^{i} a_k \times \prod_{k=i+1}^{n} a_k \]
Distribution

\[ c \times \sum_{k=m}^{n} a_k = \sum_{k=m}^{n} (c \times a_k) \]
Change of variable

\[ \sum_{k=0}^{6} \frac{1}{k+1} \]
Factorial

\[ n! = n \times (n - 1) \times (n - 2) \times \ldots \times 2 \times 1 \]
\[ 0! = 1 \]
\[ n! = n \times (n - 1)! \]