\[ a_n = a_{n-1} + n - 1 \quad a_1 = 0 \]

\[ n = 5 \]

\[ a_5 = a_4 + 4 \]

\[ = a_3 + (a_4 - 1) + 4 \]

\[ = a_3 + 3 + 4 \]

\[ = a_2 + 2 + 3 + 4 \]

\[ = a_1 + 1 + 2 + 3 + 4 \]

\[ = \sum_{i=0}^{n} i \]

\[ \sum_{i=1}^{n} i \cdot 2 \]

\[ \sum_{i=1}^{n} 2 \cdot i \]

\[ \sum_{c=1}^{n} 2 \]
\[ a \equiv b \]

\[ 4 \mid a - b \]

Reflexive: \( aRa \)

\[ a \equiv a = 4 \mid a - a = 4 \mid 0 \]

Every number including 4 divides 0

Therefore, \( a \equiv b \) is reflexive

Symmetric: \( aRb \rightarrow bRa \)

if \( aRb \) then \( bRa \)

\[ 4 \mid a - b \]

\[ \Rightarrow a - b = 4k \]

Multiply both sides by -1

\[ -b + a = 4(-k) \]

\[ \Rightarrow 4 \mid b - a \]

Here, \( a \equiv b \) and \( b \equiv 4a \)

so it is symmetric
Transitive

\[ y \text{ aRb } \land \text{ bRc } \Rightarrow \text{ aRc} \]

\[ 4 \mid a-b \]

\[ a-b = 4k \]

\[ b-c = 4l \]

Adding the two equations:

\[ a-c = 4(k+l) \quad k, l \in \mathbb{Z} \]

\[ \Rightarrow \quad 4 \mid a-c \]

Therefore it is transitive and hence equivalent.

\[
\frac{1}{2} \quad \frac{2}{4} \quad \frac{3}{6} \quad \cdots
\]
Partition

Subsets $A_1$, $A_2$, $A_3$ are partitions of set $A$ such that they are mutually disjoint and $A_1 \cup A_2 \cup A_3 = A$

$A = \{1, 2, 3, 4, 5, 6\}$

$A_1 = \{1, 2\}$, $A_2 = \{3, 4, 5\}$, $A_3 = \{6\}$

$A_3 = \{1, 6\} \times$
$R = \{ \}
\
a R a
\
There is a loop 'from every letter itself.'

Symmetric
\
a R d and d R a
\
b R f f R b
\
d R e l e d
\
a R e l e R a

There is an arrow going from each one of these letters.
\[ x = \{ a, b, c, d, e, f \} \]

Diagram:

- \( G_b \) is connected to \( a \) and \( f \).
- \( c \) is connected to \( d \) and \( e \).
- \( d \) is connected to \( e \) and \( f \).

Truth table:

- \( aRb \) is true if \( a = b \) and \( bRa \) is true if \( b = a \).
- Reflexive: \( aRa \) for all elements.
- Symmetric: \( aRb \) implies \( bRa \).
- Transitive: \( aRb \) and \( bRc \) implies \( aRc \).
\[ R = \{ (\text{Alabama}, \text{Arizona}) \} \]

\[ a R a \]

\[ \text{Alabama} R \text{ Alabama} \]

\[ \text{Alabama} R \text{ Arizona} \]

\[ \text{then Arizona} R \text{ Alabama} \]

\[ \{ i \mid a R b \land b R c \rightarrow a R c \} \]

\[
\sin(a) = \sin(b)
\]

**Reflexive**

\[
\sin(a) = \sin(a)
\]

**Symmetric**

\[
\sin(a) = \sin(b) \quad \text{Also} \quad \sin(b) = \sin(a)
\]
\[
\text{if } a \rightarrow b \text{ then } b \rightarrow a
\]

\[
a \rightarrow b \quad \rightarrow \quad b \rightarrow a
\]

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If \( \sin (a) = \sin (b) \)
\( \sin (b) = \sin (c) \)
then \( \sin (a) = \sin (c) \)

By the transitive property of equality.
Therefore it is equivalent

Anti-symmetric

\[ a R b \land b R a \rightarrow a = b \]
\[ a \neq b \land a R b \rightarrow b \not R a \]
$\geq$ over $\mathbb{Z}$

$a \geq b$

- Reflexive: $a \geq a$ — yes.

$a \geq b \rightarrow b \neq a$

- If $2 \geq 1$ then $1 \neq 2$

By the property of the greater than for integers, it is antisymmetric.

$a \geq b$ and $b \geq c \rightarrow a \geq c$

By the transitive property of inequality for integers.
\[ A = \{1, 3, 5, 6, 9\} \quad a \mid b \quad a \mid b \]
\[ R = \{ (1, 1), (1, 3), (3, 6), (3, 9), (3, 3) \} \]

Reflexive: \[ a \mid a \]

Every integer divides itself.

Symmetric: \[ a \mid b \rightarrow b \mid a \]

If \[ a \mid b \]
\[ b = ak \]
\[ a \mid \frac{b}{k}.l \]
\[ a = \frac{b}{k}l \Rightarrow kl = 1 \]

\[ a \mid Rb \forall b \mid Ra \rightarrow a = b \]
Transitive
\[ a \mid b \text{ and } b \mid c \Rightarrow a \mid c \]
\[ b = a k \]
\[ c = b \cdot k \]
\[ a \cdot c = a \cdot (a \cdot k) \]
Integers are closed in multiplication:
\[ a, 1 \mid c \]
Therefore, it is reflexive, antisymmetric and transitive.
Total — Every element in relation/set is comparable to every other element.

\[ a \leq b \text{ or } b \leq a \]

Transitive

\[ a \leq b \text{ and } a \neq b \rightarrow b \not\leq a \]

Every total order relationship is a partial order relation by every partial order relation is not a total order relation.