1. Prove each of the following claims using strong induction. Be sure to:

- Assert that you are invoking strong induction on a particular variable.
- State the element(s) for which the base case(s) apply, and prove them.
- Carefully state the inductive hypothesis. (Be sure to follow the examples from class!)
- Label the inductive step and state what you must show
- Prove the inductive step, being careful to label the point at which the inductive hypothesis is being applied.

(a) Let \( a_n \) be the recurrence defined by: \( a_0 = 12, a_1 = 29, \) and \( \forall n \geq 2, a_n = 5a_{n-1} - 6a_{n-2}. \)
Prove that \( \forall n \geq 0 : a_n = 5 \cdot 3^n + 7 \cdot 2^n \)

(b) Let \( a_n \) be the recurrence defined by: \( a_0 = 1, a_1 = 2, a_2 = 3, \) and, \( \forall n \geq 3, a_n = a_{n-1} + a_{n-2} + a_{n-3}. \)
Prove that \( \forall n \geq 0 : a_n \leq 3^n. \)

(c) Let \( a_n \) be the recurrence defined by: \( a_1 = 1, a_2 = 3, \) and, \( \forall n \geq 3, a_n = a_{n-1} + a_{n-2}. \)
Prove that \( \forall n \geq 1 : a_n \leq \left( \frac{7}{4} \right)^n. \)

(d) Prove that if you only have 3 cent and 5 cent coins available, you can generate any amount of money that is greater than or equal to 13 cents.

2. Let \( a_n \) be the recurrence defined by: \( a_0 = 4, a_1 = 7, \)
and \( \forall n \geq 2, a_n = 2a_{n-1} + 5a_{n-2}. \)
Using constructive induction, find integer constants \( A \) and \( B \) such that, \( \forall n \geq 0 : a_n \leq AB^n. \)
Try to make \( B \) as small as possible.