Homework 5 – Due **Friday 3/15**

1. What is the remainder when $7^{5555}$ is divided by 6? Explain briefly.
2. What is the remainder when $5^{7777}$ is divided by 6? Explain briefly.
3. What is the remainder when $5^{6666}$ is divided by 6? Explain briefly.
4. Let $k$ be the number of people on the planet Earth. What is the remainder when $15k^{200} + 6(k + 2)^{71} + 302$ is divided by 3? Explain briefly.
5. Use modular congruence (mod 2) to decide whether or not the following number is even or odd: $722^{77} - 333^{99}(55^{100})$
6. An “Equivalence Relation” is reflexive, symmetric, and transitive. Prove that modular congruence is an equivalence relation, by proving the following:
   a. Prove that modular congruence is reflexive: $\forall x \in \mathbb{Z}, \forall n \geq 1 [x \equiv_n x]$
   b. Prove that modular congruence is symmetric: $\forall x, y \in \mathbb{Z}, \forall n \geq 1 [x \equiv_n y \rightarrow y \equiv_n x]$
   c. Prove that modular congruence is transitive:
      $\forall x, y, z \in \mathbb{Z}, \forall n \geq 1 [(x \equiv_n y \text{ and } y \equiv_n z) \rightarrow x \equiv_n z]$
7. Prove that for all natural numbers, $n$: $n$ is not congruent to $n^2 - 4$ (mod 9)
8. Prove that for all natural numbers, $n$: $n$ is divisible by 9 if and only if the sum of the digits of $n$ is divisible by 9. Hint: We did a similar example in class.
9. Show that if $n$ is a natural number and $n$ is congruent to 3 (mod 4) then one of the prime factors of $n$ must also be congruent to 3 (mod 4).
10. Prove that for all integers $n$ and $d$: $d | n \iff n = d \cdot \lfloor n/d \rfloor$
11. Prove that for any real number $x$: If $x$ is not an integer, then $[x] + [-x] = -1$