Announcements

• Homework due tomorrow (and HW #9 will be assigned).
Permutations

Different ways of ordering objects in a list are called permutations.

Examples:

- On your vacation you will visit Germany, Italy, France, and Greece. How many different ways are there to organize this trip?
- How many ways are there to shuffle a deck of cards?

Can we derive a formula for this?

“Number of permutations of n objects” = ???

n!
Ways to select some members from a set

Suppose we have a set of 7 colors and we want to select three of them. How many ways can it be done?

It depends… 4 different scenarios:
1. No repeats; order matters (lists with no repeats)
   RGY, BGR, GYB, GBY, BGY, etc.
2. No repeats; order doesn’t matter (sets)
   RGY, BGR, GYB, etc. (Can’t also include GBY, BGY)
3. Repeats allowed; order matters (lists)
   RRY, BBB, RGG, GRG, GGR, etc.
4. Repeats allowed; order doesn’t matter (bags)
   RRY, BBB, RGG, etc. (Can’t also include GRG, GGR)
1. No repeats; order matters

Examples:
- How many ways are there to go on a trip where you visit 5 of the 50 states?
- The Kentucky Derby has 20 horses. How many outcomes (win, place, and show) are possible?

These are called **R-Permutations**

Can we derive a formula for this?

“Number of permutations of $n$ objects taken $r$ at a time” = ???

$$P(n, r) = _nP_r = \frac{n!}{(n - r)!}$$
2. No repeats; order doesn’t matter

**Examples:**
- Three senators will be chosen to form a sub-committee. How many ways can this be done?
- How many different 5-card poker hands are possible?

These are called **Combinations**. (Note that this is the same as **subsets** of a fixed size.)

**Can we derive a formula for this?**

“Number of combinations of n objects taken r at a time” = ???

\[
C(n, r) = \binom{n}{r} = \frac{P(n, r)}{r!} = \frac{n!}{(n-r)!r!}
\]
3. Repeats allowed; order matters

Examples:
- Every day the forecast is either rainy, sunny, or cloudy. How many forecasts are possible for a given week?
- How many different ways can you answer a quiz with 10 multiple choice questions, labelled A, B, C, and D?

These are called **Tuples**

Can we derive a formula for this?

“Number of ways to select an $r$-tuple from a set of size $n$” = $n^r$
4. Repeats allowed; order doesn’t matter

Examples:
- I have a bag full of: 7 snickers bars, 12 milkyway bars, and 15 KitKats. How many ways are there to reach in and grab 4 pieces of candy?
- M&M’s come in 6 colors. How many different handfuls of 25 M&M’s are possible?

These are called multi-sets.

Can we derive a formula for this?

“Number of multisets of size r taken from set of size n” = \[ \binom{n + r - 1}{r} \]
More multiset questions

Examples:

- I have 330 students. How many grade distributions are possible? (For example, a grade distribution might be: 100 A’s, 50 B’s, 100 C’s, 50 D’s, and 30 F’s.)

- How many ways are there to distribute 50 tennis balls among 4 containers. (Any number of balls could be put into any container, including 0.)
### Summary: Choosing $r$ elements out of $n$ elements

<table>
<thead>
<tr>
<th></th>
<th>order matters</th>
<th>order doesn’t matter</th>
</tr>
</thead>
<tbody>
<tr>
<td>repetition allowed</td>
<td>$n \times \cdots \times n = n^r$ ($r$ times)</td>
<td>$\binom{n+r-1}{r}$</td>
</tr>
<tr>
<td>repetition not allowed</td>
<td>$P(n, r) = \frac{n!}{(n-r)!}$</td>
<td>$\binom{n}{r} = \frac{n!}{(n-r)! r!}$</td>
</tr>
</tbody>
</table>
Another kind of Question

● Examples:
  – Arrangements of the word “mississippi”
  – Assume you have a set of 15 beads:
    • 6 green
    • 4 orange
    • 3 red
    • 2 black

How many ways are there to arrange them in a row?