Problem 1. Illustrate the operation of counting sort algorithm on the array, $A = [12, 0, 4, 0, 2, 6, 8, 12, 2, 6, 4]$. 

Problem 2. Assume you are given a set of seemingly identical keys to a treasure chest and a two pan weighing scale. One of the keys is lighter than the rest and is the one you need to open the chest. The treasure can be all yours if you can find the lightest key in the least number of weighings. If you exceed the optimal number of weighings, you will lose the treasure. Find the least number of weighings for 2, 4, 8, 16 and 32 keys. Explain your strategy.

Problem 3. Continuing with the key weighing of Problem 2, assume you are given 12 keys, except this time, instead of knowing that one of them is lighter than the rest, you are told that one key is different. That means it may be lighter or heavier than the rest.

1. What is the minimum number of weighings needed to find the different key? Show your work.

2. Use the decision tree model to find the lower bound on the number of weighings. Show your work.