

# CMSC 330 Quiz 4 Fall 2021 Solutions

## Q1. Explicit Parenthesis

Make the parentheses in the following lambda expressions explicit:

$$\lambda x. \ x \ y \ \lambda y. \ y \ y \ z$$

Note: You may use  $\lambda$ , \, or L to denote the lambda symbol.

$(\lambda x. ((x\ y)\ (\lambda y. (y\ y)\ z)))$

## Q2. Alpha Conversion

Select the valid alpha conversions of the following lambda expression:

$$(\lambda x. \ x \ (\lambda a. \ x \ a \ b)) \ y$$

$$B: (\lambda x. \ x \ (\lambda c. \ x \ c \ b)) \ y$$

C:  $(\lambda c. \ c \ (\lambda w. \ c \ w \ b)) \ y$

### Q3. Free Variables

Select the following lambda expressions that **contain** free variables.

$$A: (\lambda b. \ a \ (\lambda a. \ a) \ b)$$

$$D: (\lambda c. \lambda b. c \lambda b. a \lambda a. a) (\lambda b. b)$$

## Q4. $\beta$ -Reduction

Solve the following beta reductions. Show all steps with **explicit parentheses** to receive full points.

$$(\lambda a. \ a) \ ((\lambda x. \ \lambda y. \ x \ y) \ (\lambda b. \ b) \ (\lambda c. \ c))$$

You may solve this using either call-by-value or call-by-name.

$$\begin{aligned}
 &= (\lambda a. a) (((\lambda x. \lambda y. x y) (\lambda b. b)) (\lambda c. c)) \\
 &= (((\lambda x. \lambda y. x y) (\lambda b. b)) (\lambda c. c)) \\
 &= (\lambda y. (\lambda b. b) y) (\lambda c. c) \\
 &= (\lambda b. b) (\lambda c. c) \\
 &= (\lambda c. c)
 \end{aligned}$$

- Explicit Parenthesis
  - Using call-by-name
  - Substitute x with  $(\lambda b. b)$
  - Substitute y with  $(\lambda c. c)$
  - Substitute b with  $(\lambda c. c)$

## Q5. Call-by-Name vs. Call-by-Value

Q5.1. Show the  $\beta$ -Reduction of the expression  $(\lambda a. \lambda b. b) ((\lambda c. c) (\lambda d. d))$  using **call-by-name**. Show all steps with **explicit parentheses** for full credit.

$$= (\lambda b. \ b)$$

- Substitute a with  $((\lambda c. \ c) \ (\lambda d. \ d))$

Q5.2. Show the  $\beta$ -Reduction of the expression  $(\lambda a. \lambda b. b) ((\lambda c. c) (\lambda d. d))$  using **call-by-value**. Show all steps with **explicit parentheses** for full credit.

$$\begin{aligned}
 &= (\lambda a. \lambda b. b) (\lambda d. d) \\
 &= (\lambda b. b)
 \end{aligned}$$

- Substitute c with  $(\lambda d. d)$
  - Substitute a with  $(\lambda d. d)$