CMSC 330 Quiz 5 Spring 2022 Solutions

Q1. Lambda Calculus

Consider the following lambda expression.

\( \lambda a. \lambda b. b c \lambda c. d f a \)

**Note:** To represent \( \lambda \), you may either copy and paste the symbol \( \lambda \) or just type the characters \( L \) or \( \backslash \) in your solutions.

Q1.1. Make the parenthesis explicit

\( (\lambda a. (\lambda b. ((b c) (\lambda c. ((d f) a)))))) \)

Q1.2. Which of the following are free (or unbound) variables? Select all that apply.

- a
- b
- c
- d
- f

Q1.3. Which of the following are valid \( \alpha \)-conversions? Select all that apply.

- \( \lambda x. \lambda b. b c \lambda c. d f x \)
- \( \lambda w. \lambda b. b c \lambda c. d f a \)
- \( \lambda a. \lambda b. b x \lambda x. d f a \)
- \( \lambda a. \lambda b. b c \lambda w. d f a \)
- \( \lambda a. \lambda y. y c \lambda c. d f a \)

Q2. Concepts

For each of the questions below, select whether the given statement is true or false.

Q2.1. Reducing lambda expressions with call-by-name and call-by-value always yields the same result, assuming that all expressions terminate.

T/F

Q2.2. Reducing lambda expressions with call-by-name and call-by-value always takes the same number of steps/reductions.

T/F

Q3. Beta Reduction

Reduce the following lambda calculus expression to the \( \beta \)-normal form.

\( (\lambda y. \lambda y. y y) a (\lambda y. y) b \)

Show each step, including any \( \beta \)-reduction or \( \alpha \)-conversion. If there is infinite recursion, write "Infinite Recursion".

**Notes:**

- You must make all parenthesis explicit before reducing the expression.
- You also must perform valid \( \alpha \)-conversions to remove all ambiguity/duplicate variables.
- To represent \( \lambda \), you may either copy and paste the symbol \( \lambda \) or just type the characters \( L \) or \( \backslash \) in your solutions.
\[(\lambda y. \lambda y. y \ y) \ a \ (\lambda y. \ y) \ b\]
\[
= (((\lambda y. \ (\lambda y. \ (y \ y))) \ a) \ (\lambda y. \ y)) \ b) \quad --- \ Explicit \ Parenthesis
\]
\[
= (((\lambda y. \ (\lambda m. \ (m \ m))) \ a) \ (\lambda n. \ n)) \ b) \quad --- \ \alpha\text{-Conversion}
\]
\[
= (((\lambda m. \ (m \ m)) \ (\lambda n. \ n)) \ b)
\]
\[
= ((\lambda n. \ n) \ b)
\]
\[
= b
\]

Q4. Mystery Operator

Suppose we have a mystery lambda expression \textit{mys} such that for any input \(x, a\), we have the following:

\[
\texttt{mys} \ x \ a = x
\]

\textbf{Note:} To represent \(\lambda\), you may either copy and paste the symbol \(\lambda\) or just type the characters \texttt{L} or \texttt{\}\ in your solutions.

Q4.1. Give a possible lambda expression for \textit{mys}.

\[
\lambda x. \, \lambda y. \, x
\]

\textit{Answers with form} \(\lambda a. \, \lambda b. \, x\) \textit{will not receive partial credit.}

Q4.2. Using the expression from \textbf{Q4.1}, reduce the following expression to the \(\beta\)-normal form.

\[
\texttt{mys} \ (\lambda x. \ x)
\]

Show each step, including any \(\beta\)-reduction or \(\alpha\)-conversion. If there is infinite recursion, write "Infinite Recursion".

\[
\texttt{mys} \ (\lambda x. \ x)
\]
\[
= (\lambda x. \, \lambda y. \, x) \ (\lambda x. \ x)
\]
\[
= (\lambda y. \ (\lambda x. \ x))
\]

\textbf{Even if expression in Q4.1 was incorrect, we have given partial credit for correct \(\beta\)-reduction.}