Node types:

- **2-Node**
  - 1 key
  - 2 children

- **3-Node**
  - 2 keys
  - 3 children

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**Recap:**

- **AVL:** Height balanced
- **Binary:**
- **2-3 Tree:** Height exact
- **Variable Width**

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**Adoption (Key Rotation):**

1 + 3 = 2 + 2

**Merge:**

1 + 2 / 2 + 1 → 3

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**Def:** A 2-3 tree of height $h$ is either:

- Empty ($h = -1$)
- A 2-Node root and two subtrees, each 2-3 tree of height $h - 1$
- A 3-Node root and three subtrees ... height $h - 1$

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**Thm:** A 2-3 tree of $n$ nodes has height $O(\log n)$

**Roughly:** $\log_3 n \leq h \leq \log_2 n$

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**Example:**

2-3 tree of height 2

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**How to maintain balance?**

- Split
- Merge
- Adoption (Key rotation)

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**Conceptual tool:**

We'll allow 1-nodes

+ 4-nodes temporary

1-node

A

A C E G
Insertion example: insert(6)

Dictionary operations:
- **Find**: straightforward
- **Insert**: find leaf node where key "belongs" and add it (may split)
- **Delete**: find/replacement/merge or adopt

Implementation?

2-3 Trees II

Delete Example: delete(5)

Deletion remedy:
- Have a 3-node neighboring sibling : adopt
- O.w.: Merge with either sibling + steal key from parent

Example (continued)
Encoding 3-node as binary tree node

Some history:

2-3 Trees: Bayer 1972
Red-black Trees: Guibas & Sedgewick 1978 (a binary variant of 2-3)

Rumor - Guibas had two pens: red & black to draw with

Red-Black and AA-Trees

AA-Trees: Simpler to code
- No null pointers: Create a sentinel node, nil, and all nulls point to it → nil
- No colors: Each node stores one level number: Red child is at same level as parent.

What we need are stricter rules!

AA-Tree:
Arne Anderson 1993
New rule:
6. Each red node can arise only as right child (of a black node)

Rules:
1. Every node labeled red/black
2. Root is black
3. Nulls treated as if black
4. If node is red, both children are black
5. Every path, from root to null has same no. of black

Lemma: A red-black tree with n keys has height \( O(\log n) \)
Proof: It's at most twice that of a 2-3 tree.

Q: Is every Red-Black Tree the encoding of some 2-3 tree?

Nope! Alternatives that satisfy rules:

A "left-skewed" encoding corresponds to 2-3-4 trees
Announcements - 02/28

- Prog Assign 1 - Due tomorrow 11:59pm
  → Skeleton code was updated
  → test01-input.txt
  → test01a-input
  → b-input

- Homework 2 - Binary Search Trees
  → Due Tue, Mar 7, start of class

- Midterm 1 - Next Thu, Mar 9 in class
  - Closed-book / Closed-notes
  - 1 "cheat sheet" - front & back

- Practice Problems - Coming