Summer 2003 **CMSC 351: Homework 1** Clyde Kruskal

Due at the start of class Friday, June 6, 2003.

Problem 1. Use mathematical induction to show that

$$(a)\quad \sum_{i=1}^{n} i(i+1) = \frac{n(n+1)(n+2)}{3} \qquad\qquad (b)\quad \sum_{i=0}^{n} 2^i = 2^{n+1} - 1$$

Problem 2. See bottom of page 53 of CLRS (bottom of page 34 in CLR) and/or other side of this sheet.

(a) Assume $b^x = a$. What is x (in terms of a and b)?

(b) Using only part (a), show that $\log_c(ab) = \log_c a + \log_c b$.

(c) Show that $a^{\log_b n} = n^{\log_b a}$

Problem 3. Differentiate the following functions:

(a) $\ln(x^2 + 5)$

(b) $\lg(x^2 + 5)$

(c) $\frac{1}{\ln(x^2+5)}$

Problem 4. Integrate the following functions:

(a) $\frac{1}{x}$

(b) $\frac{1}{3x+7}$

(c) $\ln x$ [HINT: Use integration by parts.]

(d) $x \ln x$ [HINT: Use integration by parts.]

(e) $x \lg x$

Problem 5. Consider the problem of not only finding the value of the maximum contiguous sum in an array, but also determining the two endpoints. Give a linear time algorithm for solving this problem. [What happens if all entries are negative?]

Problem 6. We can generalize the "maximum contiguous sum problem" to two dimensions: Given an $m \times n$ array of (positive and negative) numbers, find the largest sum of values in a (contiguous) rectangle.

(a) Write down an English description of the "brute force" algorithm for the "maximum contiguous rectangle problem". One or two sentences should suffice.

(b) Write down the "brute force" algorithm in psuedocode.

(c) How many times is the inner loop executed? Write it using summations.

(d) Simplify your answer. Justify your work. [If you do this right, the solution involves very little calculation.]

(e) **Challenge Problem.** Find a better algorithm for the "maximum contiguous rectangle problem". How well can you do?

$$\begin{aligned}
\lg n &= \log_2 n \\
\ln n &= \log_e n \\
\lg^k n &= (\lg n)^k \\
\lg \lg n &= \lg(\lg n)
\end{aligned}$$

For all real $a > 0$, $b > 0$, $c > 0$, and n,

$$\begin{aligned}
a &= b^{\log_b a} \\
\log_c(ab) &= \log_c a + \log_c b \\
\log_b a^n &= n \log_b a \\
\log_b a &= \frac{\log_c a}{\log_c b} \\
\log_b(1/a) &= -\log_b a \\
\log_b a &= \frac{1}{\log_a b} \\
a^{\log_b n} &= n^{\log_b a}
\end{aligned}$$