CMSC 132:
Object-Oriented Programming II

Graphs & Graph Traversal

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Graph Data Structures

- Many-to-many relationship between elements
  - Each element has multiple predecessors
  - Each element has multiple successors
Graph Definitions

Node
- Element of graph
- State
  - List of adjacent nodes

Edge
- Connection between two nodes
- State
  - Endpoints of edge
Graph Definitions

- Directed graph
  - Directed edges
- Undirected graph
  - Undirected edges
Graph Definitions

- Weighted graph
  - Weight (cost) associated with each edge
Graph Definitions

Path

- Sequence of nodes $n_1, n_2, \ldots, n_k$
- Edge exists between each pair of nodes $n_i, n_{i+1}$

Example

- A, B, C is a path
- A, E, D is not a path
Graph Definitions

- **Cycle**
  - Path that ends back at starting node
  - Example
    - A, E, A
    - A, B, C, D, E, A

- **Simple path**
  - No cycles in path

- **Acyclic graph**
  - No cycles in graph
Graph Definitions

- **Reachable**
  - Path exists between nodes

- **Connected graph**
  - Every node is reachable from some node in graph

Unconnected graphs
Graph Operations

Traversal (search)

- Visit each node in graph exactly once
- Usually perform computation at each node
- Two approaches
  - Breadth first search (BFS)
  - Depth first search (DFS)
Breadth-first Search (BFS)

Approach

- Visit all neighbors of node first
- View as series of expanding circles
- Keep list of nodes to visit in queue

Example traversal

1. n
2. a, c, b
3. e, g, h, i, j
4. d, f
Breadth-first Tree Traversal

Example traversals starting from 1

Left to right  Right to left  Random
Traversals Orders

Order of successors
- For tree
  - Can order children nodes from left to right
- For graph
  - Left to right doesn’t make much sense
  - Each node just has a set of successors and predecessors; there is no order among edges

For breadth first search
- Visit all nodes at distance k from starting point
- Before visiting any nodes at (minimum) distance k+1 from starting point
Depth-first Search (DFS)

**Approach**
- Visit all nodes on path first
- **Backtrack** when path ends
- Keep list of nodes to visit in a stack

**Example traversal**
1. N
2. A
3. B, C, D, ...
4. F…
Depth-first Tree Traversal

Example traversals from 1 (preorder)

Left to right

Right to left

Random
Traversals Algorithms

Issue
- How to avoid revisiting nodes
- Infinite loop if cycles present

Approaches
- Record set of visited nodes
- Mark nodes as visited
Traversals – Avoid Revisiting Nodes

- Record set of visited nodes
  - Initialize { Visited } to empty set
  - Add to { Visited } as nodes is visited
  - Skip nodes already in { Visited }

\[
\begin{align*}
V &= ∅ \\
V &= \{ 1 \} \\
V &= \{ 1, 2 \}
\end{align*}
\]
Traversal – Avoid Revisiting Nodes

Mark nodes as visited
- Initialize tag on all nodes (to False)
- Set tag (to True) as node is visited
- Skip nodes with tag = True
Traversing Algorithm Using Sets

\{ \text{Visited} \} = \emptyset

\{ \text{Discovered} \} = \{ \text{1st node} \}

\text{while } ( \{ \text{Discovered} \} \neq \emptyset )

\quad \text{take node } X \text{ out of } \{ \text{Discovered} \}

\quad \text{if } X \text{ not in } \{ \text{Visited} \}

\quad \quad \text{add } X \text{ to } \{ \text{Visited} \}

\quad \text{for each successor } Y \text{ of } X

\quad \quad \text{if ( } Y \text{ is not in } \{ \text{Visited} \} \text{ )}

\quad \quad \quad \text{add } Y \text{ to } \{ \text{Discovered} \}
Traversal Algorithm Using Tags

for all nodes X

set X.tag = False

{ Discovered } = { 1st node }

while ( { Discovered } ≠ ∅ )

    take node X out of { Discovered }

    if (X.tag = False)

        set X.tag = True

        for each successor Y of X

            if (Y.tag = False)

                add Y to { Discovered }
BFS vs. DFS Traversal

- Order nodes taken out of \{ Discovered \} key
- Implement \{ Discovered \} as Queue
  - First in, first out
  - Traverse nodes breadth first
- Implement \{ Discovered \} as Stack
  - First in, last out
  - Traverse nodes depth first
BFS Traversal Algorithm

for all nodes X
    X.tag = False
put 1st node in Queue
while ( Queue not empty )
    take node X out of Queue
    if (X.tag = False)
        set X.tag = True
        for each successor Y of X
            if (Y.tag = False)
                put Y in Queue
DFS Traversal Algorithm

for all nodes X
   X.tag = False

put 1st node in Stack

while (Stack not empty )
   pop X off Stack
   if (X.tag = False)
      set X.tag = True
      for each successor Y of X
         if (Y.tag = False)
            push Y onto Stack
Recursive Graph Traversal

- Can traverse graph using recursive algorithm
  - Recursively visit successors

Approach
- Visit (X)
  - for each successor Y of X
    - Visit (Y)

- Implicit call stack & backtracking
  - Results in depth-first traversal
Recursive DFS Algorithm

Traverse()

for all nodes X
set X.tag = False

Visit (1st node)

Visit (X)

set X.tag = True

for each successor Y of X

if (Y.tag = False)

Visit (Y)