Summer 2007 **CMSC 351: Homework 1** Clyde Kruskal

Due at the start of class Friday, June 8, 2007.

Problem 1. Consider a tree where every node has exactly zero or two children. Let I be the number of internal nodes (nonleaves) and L be the number of leaves.

(a) Give and equation relating I and L. (I.e., how does the number of internal nodes compare with the number of leaves?)

(b) Prove your answer using mathematical induction.

Problem 2. Consider the problem of not only finding the value of the maximum contiguous sum in an array, but also determining the two endpoints. Give a linear time algorithm for solving this problem. [What happens if all entries are negative?]

Problem 3. We can generalize the "maximum contiguous sum problem" to two dimensions to solve the "maximum contiguous rectangle problem". Given an $m \times n$ array of (positive and negative) numbers, find the largest sum of values in a (contiguous) rectangle.

(a) Write down an English description of the "brute force" algorithm for the "maximum contiguous rectangle problem". One or two sentences should suffice.

(b) Write down the "brute force" algorithm in psuedocode.

(c) How many times is the inner loop executed? Write it using summations.

(d) Simplify your answer. Justify your work. [If you do this right, the solution involves very little calculation.]

(e) **Challenge Problem.** Find a better algorithm for the maximum contiguous rectangle problem. How well can you do?

Problem 4. Give correct pseudo-code for the modified bubble sort algorithm. Make your algorithm as "elegant" as possible.