Problem 1. Use mathematical induction to show that

(a) \[ \sum_{i=1}^{n} i(i+1) = \frac{n(n+1)(n+2)}{3} \]

(b) \[ \sum_{i=0}^{n} 2^i = 2^{n+1} - 1 \]

Problem 2. See bottom of page 53 of CLRS (bottom of page 34 in CLR) and/or the bottom of this sheet.

(a) Assume \( b^x = a \). What is \( x \) (in terms of \( a \) and \( b \))?

(b) Using only part (a), show that \( \log_c(ab) = \log_c a + \log_c b \).

(c) Show that \( a^{\log_b n} = n^{\log_b a} \)

Problem 3. Differentiate the following functions:

(a) \( \ln(x^2 + 5) \)

(b) \( \log(x^2 + 5) \)

(c) \( \frac{1}{\ln(x^2+5)} \)

Problem 4. Integrate the following functions:

(a) \( \frac{1}{x} \)

(b) \( \frac{1}{3x+7} \)

(c) \( \ln x \) \[ \text{HINT: Use integration by parts.} \]

(d) \( x \ln x \) \[ \text{HINT: Use integration by parts.} \]

(e) \( x \log x \)

\[
\begin{align*}
\lg n &= \log_2 n \\
\ln n &= \log_e n \\
\lg^k n &= (\lg n)^k \\
\lg \lg n &= \lg(\lg n)
\end{align*}
\]

For all real \( a > 0, b > 0, c > 0, \) and \( n, \)

\[
\begin{align*}
a &= b^{\log_b a} \\
\log_c(ab) &= \log_c a + \log_c b \\
\log_b a^n &= n \log_b a \\
\log_b a &= \frac{\log_c a}{\log_c b} \\
\log_b(1/a) &= -\log_b a \\
\log_b a &= \frac{1}{\log_a b} \\
a^{\log_b n} &= n^{\log_b a}
\end{align*}
\]