CMSC 351: Practice Questions for Midterm Exam

These are practice problems for the upcoming midterm exam. You will be given a sheet of notes for the exam. Also, go over your homework assignments. Warning: This does not necessarily reflect the length, difficulty, or coverage of the actual exam.

**Problem 1.** Do Problems 3.1-2 and 3.1-4 on page 50 of CLRS.

**Problem 2.** Show that \( n^2 - 7n + 1 = \Theta(5n^2 + 6n - 10) \) using the definition of \( \Theta \) given in class (do not use limits).

**Problem 3.** Use the iteration method to solve the following recurrences. You may assume \( n \) is “nice”. Prove your answers using mathematical induction.

(a) \( T(n) = 2T(n/2) + n^3, T(1) = 1. \)

(b) \( T(n) = T(\sqrt{n}) + 1, T(2) = 1. \)

(c) \( T(n) = 2T(n/2) + n \log n, T(1) = 1. \)

(d) \( T(n) = T(n - 3) + 5, T(1) = 2. \)

**Problem 4.** Which of the above problems can be solved using the “Master Theorem” derived in class. Solve them exactly using the “Master Theorem”.

**Problem 5.** Show that

(a) \( \frac{1}{2} \leq \sum_{j=1}^{\infty} \frac{1}{j^{2j}} \leq 1 \)

(b) \( 1 \leq \sum_{j=1}^{\infty} \frac{1}{j^2} \leq 2 \)

**Problem 6.** How many times on average must you flip six fair coins before you obtain exactly three heads and three tails?

**Problem 7.** Do Exercise 6.5-8 on page 142 of CLRS.

**Problem 8.** Do Problem 7-5 on pages 162-163 of CLRS.