Overview

Linear data structures
- General properties

Implementations
- Array
- Linked list

Restricted abstractions
- Stack
- Queue
Linear Data Structures

- 1-to-1 relationship between elements
- Each element has unique predecessor & successor
- Results in total ordering over elements
- For any two distinct elements $x$ and $y$, either $x$ comes before $y$ or $y$ comes before $x$
Linear Data Structures

Terminology
- Head (first element in list) ⇒ no predecessor
- Tail (last element in list) ⇒ no successor

Operations
- Add element
- Remove element
- Find element
Add & Remove Elements

Add an element

Where?
- At head (front) of list
- At tail (end) of list
- After a particular element

Remove an element

- Remove first element
- Remove last element
- Remove a particular element (e.g., String “Happy”)
  - What if “Happy” occurs more than once in list?
Accessing Elements

How do you find an element?

- At head (front) of list
- At tail (end) of list
- By position
  - Example: the 5th element
- By iterating through the list, and using relative position
  - Next element (successor)
  - Previous element (predecessor)
List Implementations

Two basic implementation techniques for lists

- Store elements in an array

- Store as a linked list
  - Place each element in a separate object (node)
  - Node contains reference to other node(s)
  - Link nodes together
Linked List

Properties

- Elements in linked list are **ordered**
- Element has successor

State of List

- Head
- Tail
- Cursor (current position)
Array Implementations

**Advantages**
- Can efficiently access element at any position
- Efficient use of space
  - Space to hold reference to each element

**Disadvantages**
- Expensive to grow / shrink array
  - Can amortize cost (grow / shrink in spurts)
- Expensive to insert / remove elements in middle
- Tricky to insert / remove elements at both ends
Linked Implementation

Advantages

- Can efficiently insert / remove elements anywhere

Disadvantages

- Cannot efficiently access element at any position
  - Need to traverse list to find element
- Less efficient use of space
  - 1-2 additional references per element
Efficiency of Operations

**Array**
- Insertion / deletion = $O(n)$
- Indexing = $O(1)$

**Linked list**
- Insertion / deletion = $O(1)$
- Indexing = $O(n)$
Linked List – Insert (After Cursor)

1. Original list & new element temp

2. Modify `temp.next` → `cursor.next`
Linked List – Insert (After Cursor)

3. Modify `cursor.next` → `temp`

4. Modify `cursor` → `temp`
1. Find before such that before.next = cursor

2. Modify before.next → cursor.next
Linked List – Delete (Cursor)

3. Delete cursor

4. Modify cursor → before.next
Doubly Linked List

Linked list where
- **Element has predecessor & successor**

Issues
- Easy to find preceding / succeeding elements
- Extra work to maintain links (for insert / delete)
- More storage per node
Doubly Linked List – Insertion

Example

Must update references in both predecessor and successor nodes
Node Structures for Linked Lists

- **Linked list**
  
  ```java
  Class Node {
    Object data;
    Node next;
  }
  ```

- **Doubly linked list**
  
  ```java
  Class Node {
    Object data;
    Node next;
    Node previous;
  }
  ```
Restricted Abstractions

Restricting the operations an abstraction supports can be a good thing

- Efficiently supporting only a few operations efficiently is easier
- If limited abstraction is sufficient, easier to reason about limited abstraction than a more general one

Restricted list abstractions

- Stack (aka LIFO queue)
- Queue (aka FIFO queue)
- Dequeue (aka double ended queue)
Stack

Stack operations

- **Push** = add element (to top)
- **Pop** = remove element (from top)

Example

(a) A three-element stack  
(b) After a `pop()` operation  
(c) After a `push(W)` operation
Stack

Properties
- Elements removed in opposite order of insertion
- Last-in, First-out (LIFO)

A restricted list where
- Access only to elements at one end
- Can add / remove elements only at one end
Stack Applications

- **Run-time procedure information**
  
  (a) Example of nested procedure calls
  
  (b) Run-time stack while in procedure D

- **Arithmetic computations**
  - Postfix notation

- **Simplified instruction set**
  - Java bytecode
Stack Implementations

- **Linked list**
  - Add / remove from head of list

  ![Diagram showing a linked list with elements X, Y, Z, and head Z, Y, X with top Z]

  (a) Logical view of the stack
  (b) Its linked list implementation

- **Array**
  - Increment / decrement Top pointer after push / pop

  ![Diagram showing an array with elements X, Y, Z, and a top pointer]

  X Y Z ...
Queue

Queue operations

- Enqueue = add element (to back)
- Dequeue = remove element (from front)

Example

(a) Three-element queue

(b) After deletion of X

(c) After insertion of W
Queue

Properties
- Elements removed in order of insertion
- First-in, First-out (FIFO)

A restricted list where
- Access only to elements at beginning / end of list
  - Add elements only to end of list
  - Remove elements only from front of list
- Alternatively, can add to front & remove from end
Queue Applications

Examples

- Songs to be played
- Jobs to be printed
- Customers to be served
- Citizens to cast votes

South Africa, 2004
Queue Implementations

- **Linked list**
  - Add to tail (back) of list
  - Remove from head (front) of list

```
front → 5 → 17 → 21 → 9
```

- **Array**

- **Circular array**
Queue – Array

- Store queue as elements in array

Problem

- Queue contents move ("inchworm effect")

As result, can not add to back of queue, even though queue is not full
Queue – Circular Array

Circular array (ring)
- \( q[0] \) follows \( q[\text{MAX} - 1] \)
- Index using \( q[i \% \text{MAX}] \)

Problem
- Detecting difference between empty and nonempty queue
Queue – Circular Array

Approach 1
- Keep Front at first in
- Keep Back at last in

Problem
- Empty queue identical to queue with 1 element
Queue – Circular Array

Approach 2

- Keep Front at first in
- Keep Back at last in – 1

Problem

- Empty queue identical to full queue
Queue – Circular Array

Inherent problem for queue of size $N$

- Only $N$ possible (Front – Back) pointer locations
- $N+1$ possible queue configurations
  - Queue with 0, 1, … $N$ elements

Solutions

- Maintain additional state information
  - Use state to recognize empty / full queue
- Examples
  - Record Size
  - Record QueueEmpty flag
- Leave empty element in queue
- Store marker in queue