Due at the start of class Wednesday, June 17, 2009.

**Problem 1.** Consider the following recurrence, defined for \( n \) a power of 5:

\[
T(n) = \begin{cases} 
19 & \text{if } n = 1 \\
3T(n/5) + n - 4 & \text{otherwise}
\end{cases}
\]

(a) Solve the recurrence exactly using the iteration method. Simplify as much as possible.

(b) Use mathematical induction to verify your solution.

**Problem 2.** Use the formulas derived in class to obtain exact solutions to the following two recurrences.

(a) Let \( n \) be a power of 2.

\[
T(n) = \begin{cases} 
4 & \text{if } n = 1 \\
5T(n/2) + 3n^2 & \text{otherwise}
\end{cases}
\]

(b) Let \( n \) be a power of 4.

\[
T(n) = \begin{cases} 
3 & \text{if } n = 1 \\
2T(n/4) + 4n + 1 & \text{otherwise}
\end{cases}
\]

**Problem 3.** Consider the following recurrence.

\[
T(n) = \begin{cases} 
0 & \text{if } n = 0 \\
T([n/2]) + T([n/4]) + 3n & \text{otherwise}
\end{cases}
\]

Use constructive induction to find a constant \( c \) such that \( T(n) \leq cn \).

**Problem 4.** Do Problem 4-2 on page 85 of CLRS.